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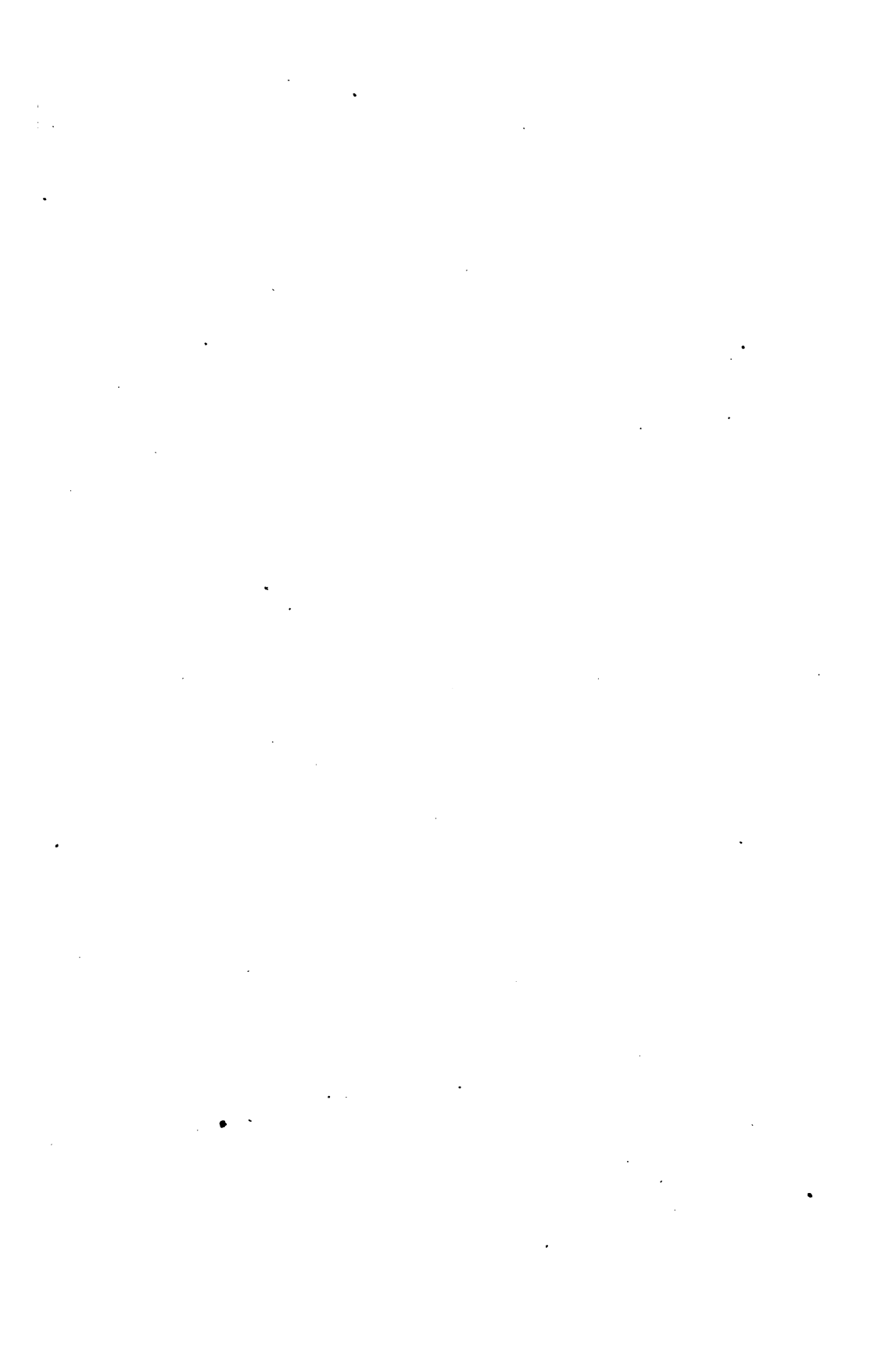
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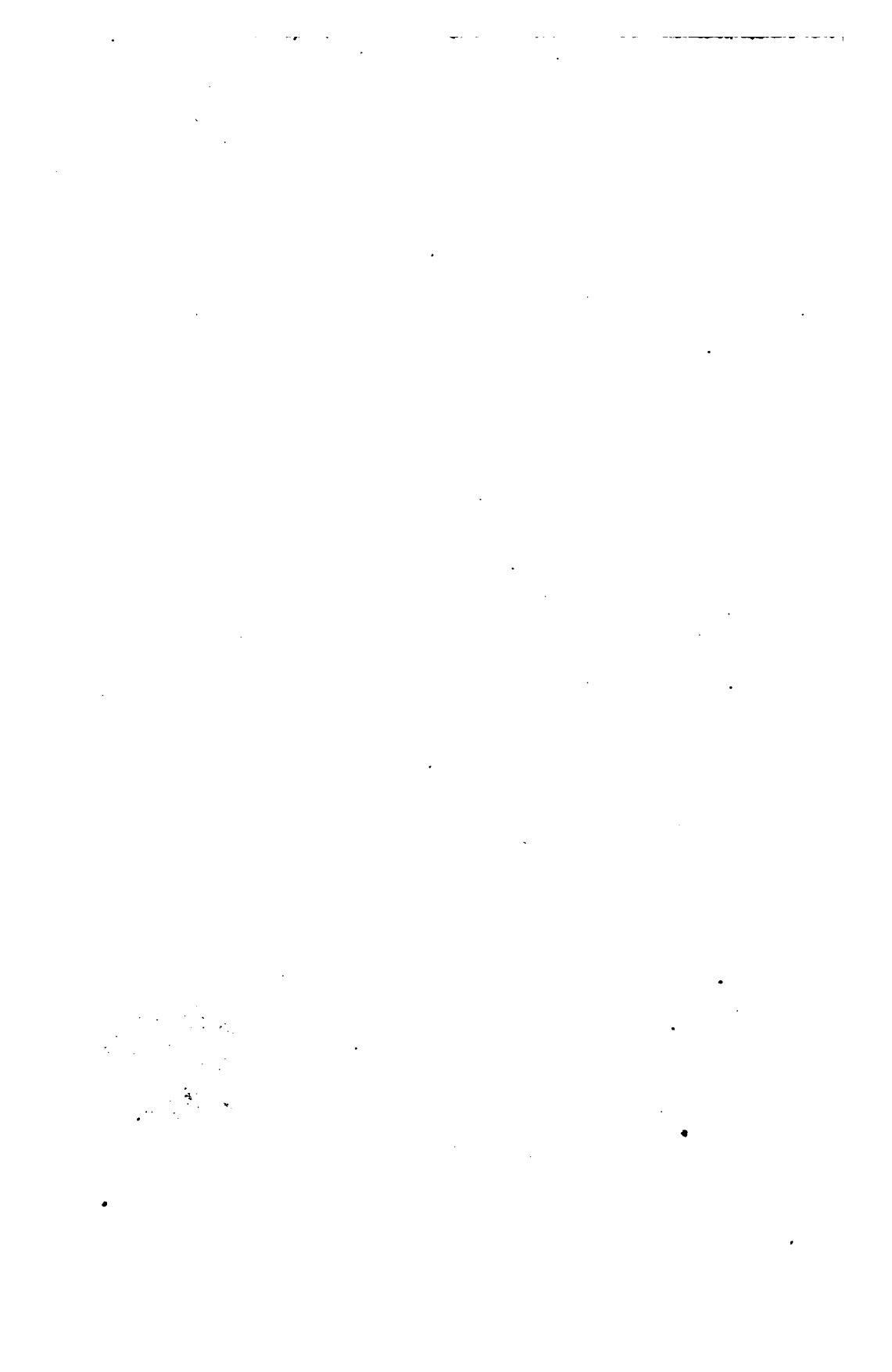
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ACCORDING TO THE TEXT OF SIMSON:

WITH ADDITIONAL FIGURES, NOTES, EXPLANATIONS, AND  
DEDUCTIONS,

BY

NICHOLAS POCOCK, M.A.

LATE MICHEL FELLOW OF QUEEN'S COLLEGE,  
AND SEVERAL TIMES ONE OF THE PUBLIC EXAMINERS IN THE  
UNIVERSITY OF OXFORD.



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SPOTTISWOODES and SHAW,  
New-street-Square.

## P R E F A C E.

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SOME apology seems to be required for sending out another edition of EUCLID'S ELEMENTS, in addition to the number which have been already published. And the Editor hopes he may lay claim to some indulgence on the score of his having attempted to make the subject more interesting than hitherto. Any one who has been occupied in teaching this subject, must have observed how difficult it is to keep the learner's attention alive during the whole of a demonstration. It is, indeed, this difficulty of fixing the attention on a subject, which has at first little or nothing that is attractive, that has deterred many from proceeding with this branch of study, and has led to the belief in some, that particular powers of mind, which they do not themselves possess, are required for its comprehension. There cannot be a greater mistake than this: every one who has any power of understanding has powers enough for this subject, provided he has the power of giving his attention to it. And the student who is really anxious to become acquainted with Geometry, may promise himself success if he will only attend to the directions hereafter given, and follow them implicitly.

Attempts have been made to obviate this difficulty by introducing symbolical notation; but this is only remedying one evil by introducing another. This

method dispenses with the necessity of fixed attention, and thus leaves the learner without the advantages which the formation of the habit of attention secures. And accordingly "Symbolical Euclids" are but little used in schools or universities, the chief objection to them appearing to be, that the learner admits the logical consequences of one step from another, without having either premise or conclusion fixed in his memory, and proceeds through the greater part of a proposition without once looking at his figure.

The careful and elaborate edition of Euclid, published by Mr. Potts at Cambridge, seven years ago, seems to obviate much of this difficulty, possessing as it does the advantage of the method of symbolical notation without its disadvantages. One of the principal features of this edition was the arrangement of the distinct statements in a proposition, in separate lines; and, in this respect, the present edition proceeds nearly on the same plan with that. What seemed wanting in that edition, the present attempts to supply, viz. the practice in drawing the figure correctly, and the direction how to do so. It may be thought, perhaps, that the directions are sometimes minute, to the extent of being trifling; and that it was scarcely necessary to add anything, in this respect, to what the original text contained. The best answer to this would be furnished by testing the powers of any learner, however well he may be thought to understand his subject, in drawing the figures of the XLIVth or XLVth Propositions of the First Book. With regard to other explanations, to which the same remark might seem

applicable, the Editor has only to observe, that it seemed worth while, at the expense of a few additional remarks and figures, to insure, if possible, a greater degree of accuracy. He will only add as a further excuse for such minuteness, that he has been often accustomed, in public examinations, to such definitions as the following :

A *straight* line is length without breadth.

A circle is a plain figure contained by one *straight* line, &c.

And most of these faults arise from want of attention to the figures. The great number of additional figures, and the large type in which the work has been printed, for the sake of rendering it more easy for the beginner to follow the arguments, have necessarily increased the expense of the publication ; but it is hoped the advantages to be drawn from this will more than compensate for it. With regard to the few Deductions which have been added, it has been kept in view to interest, and to avoid frightening the learner. Few students, excepting those who have advanced further, can do such Deductions as are usually added to editions of Euclid ; and the consequence is, that most become disheartened at finding themselves unequal to what appears to be expected of them. In the present edition, the plan has been followed of giving directions, such as will enable any learner to go through them all if he will take the trouble to refer to the Propositions indicated. Those selected are such as appeared to be most easy and interesting. It only remains to notice that the work is intended for the use of students at Oxford in their first year, and for such as are pre-

paring for that University. It contains all that is required for the examination at Responsions. If the student does not intend to pursue the subject further, he may omit the Deductions. The additional matter by the present Editor has been included in brackets, that the student may be able to distinguish what is absolutely required of him, from what is only intended to help him in acquiring that knowledge. It is very desirable that every student entering at Oxford should have read over these books at least once.

5. Lower Crescent, Clifton,  
May, 1852.



## DIRECTIONS TO THE LEARNER.

---

*First.* Procure a case of instruments.

*Secondly.* Draw each figure correctly as the construction in the Proposition directs.

*Thirdly.* In subsequent cases of having to draw the same figure, use the shorter method (if any) pointed out in the Note which follows the Proposition.

*Fourthly.* If possible, go over the first five Propositions of the First Book, *vivâ voce*, with a teacher.

*Fifthly.* Read the First Book twice before proceeding to the Second.

THE  
ELEMENTS OF EUCLID.

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BOOK I.  
DEFINITIONS.

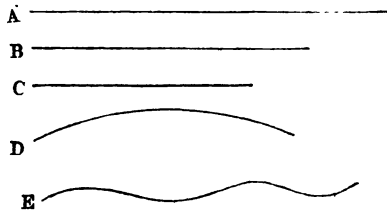
I.

A POINT is that which hath no parts, or which hath no BOOK I.  
magnitude.

A      B      C      D  
·      ·      ·      ·

II.

A line is length without breadth.



III.

The extremities of a line are points.

IV.

A straight line is that which lies evenly between its extreme points.

[A, B, C, above, are straight lines ; D, E are not straight lines, but curves.]

## BOOK I

## V.

A superficies is that which hath only length and breadth.

Fig. 1.

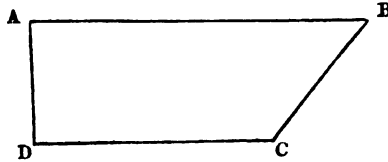
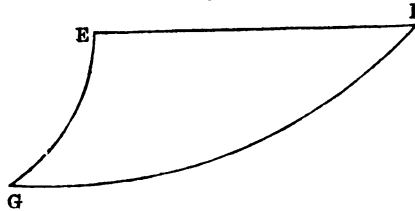


Fig. 2.



## VI.

The extremities of a superficies are lines.

[The superficies, *fig. 1.*, is bounded by four straight lines ; that in *fig. 2.* by one straight line and two lines which are not straight lines, but curves.]

## VII.

A plane superficies is that in which any two points being taken, the straight line between them lies wholly in that superficies.

## VIII.

Fig. 1.

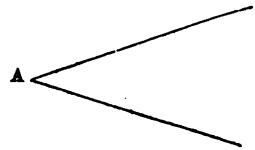
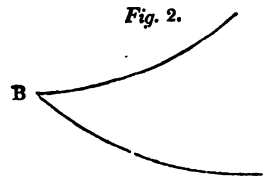


Fig. 2.

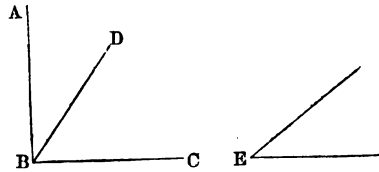


“A plane angle is the inclination of two lines to one another in a plane, which meet together, but are not in the same direction.”

[*Fig. 1.*, plane rectilinear angle ;  
*fig. 2.*, plane angle, not rectilinear.]

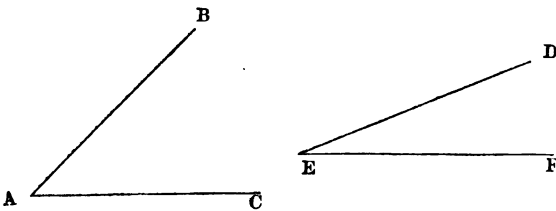
IX.

A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.



N. B. "When several angles are at one point B, any one of them is expressed by three letters, of which the letter that is at the vertex of the angle, that is, at the point in which the straight lines that contain the angle meet one another, is put between the other two letters, and one of these two is somewhere upon one of those straight lines, and the other upon the other line. Thus the angle which is contained by the straight lines AB, CB, is named the angle ABC, or CBA; that which is contained by AB, BD, is named the angle ABD, or DBA; and that which is contained by BD, CB, is called the angle DBC, or CBD; but if there be only one angle at a point, it may be expressed by a letter placed at that point; as the angle at E."

[Observe that BAC is a greater angle than DEF,



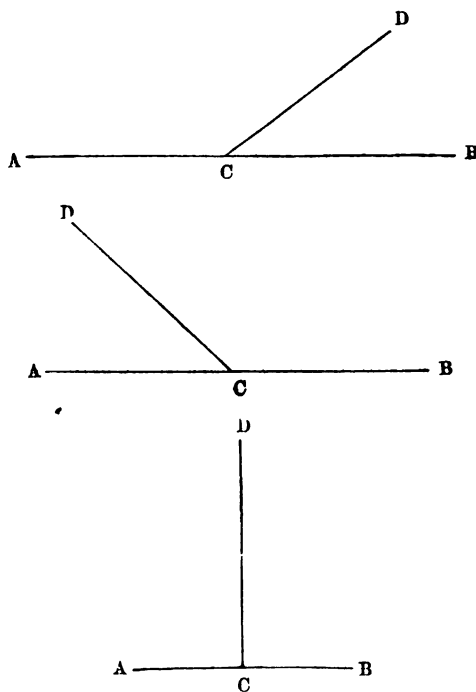
because the inclination of BA to AC is greater than that of DE to EF. This may be seen by placing EF upon AC, so that the point E shall coincide with A. Observe that DE will fall nearer to AC or EF than BA does.]

BOOK I.

## X.

When a straight line standing on another straight line makes the adjacent angles equal to one another, each of the angles is called a right angle; and the straight line which stands on the other is called a perpendicular to it.

[That there may be two such angles equal, may be exhibited by the following figures :



DC meets the right line ACB, and makes two angles DCB and DCA.

Suppose DC to be moveable round the fixed point C in the plane of the paper, and from being coincident with CB, gradually to move round till it coincides with AC. The angle DCB, which is at first less than DCA, becomes after some

time greater than it, and the increase of the one and the decrease of the other being perfectly gradual, there is some place at which the angle DCB has become not less than DCA, nor greater than it; or, in other words, equal to it.] BOOK I.

XI.

An obtuse angle is that which is greater than a right angle.



XII.

An acute angle is that which is less than a right angle.

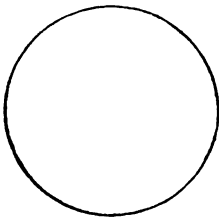
XIII.

“A term or boundary is the extremity of any thing.”

XIV.

A figure is that which is inclosed by one or more boundaries.

*Fig. 1.*



*Fig. 2.*



*Fig. 3.*

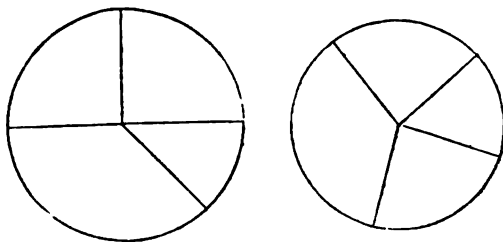
[*Fig. 1.* a figure contained by one boundary.

*Fig. 2.* and *fig. 3.* figures contained by more than one boundary.]

XV.

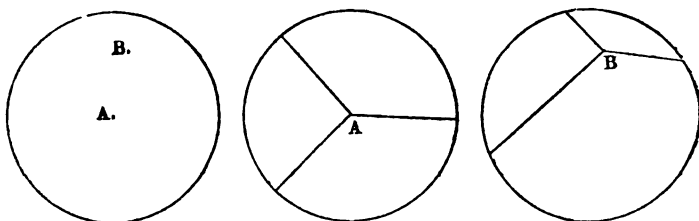
A circle is a plane figure contained by one line, which is called the circumference, and is such that all straight lines

BOOK I. drawn from a certain point within the figure to the circumference, are equal to one another.



XVI.

And this point is called the centre of the circle.



[In the above figures A is the centre of the circle ; B is not the centre.]

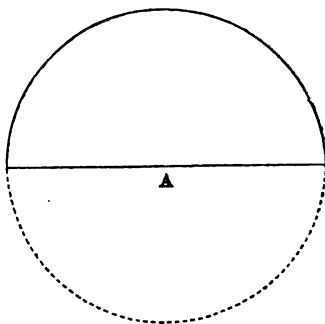
XVII.

A diameter of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

XVIII.

A semicircle is the figure contained by a diameter and the part of the circumference cut off by the diameter.

[A is called the centre of the semicircle as well as of the circle.]



XIX.

“ A segment of a circle is the figure contained by a straight line, and the circumference it cuts off.”

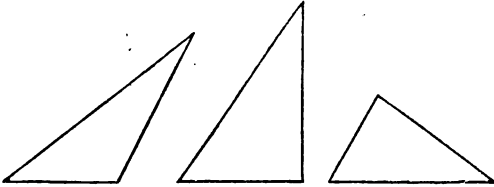
XX.

BOOK I.

Rectilineal figures are those which are contained by straight lines.

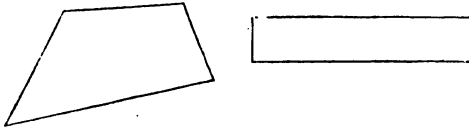
XXI.

Trilateral figures, or triangles, by three straight lines.



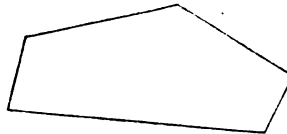
XXII.

Quadrilateral, by four straight lines.



XXIII.

Multilateral figures, or polygons, by more than four straight lines.



XXIV.

Of three-sided figures, an equilateral triangle is that which has three equal sides.

XXV.

An isosceles triangle is that which has two sides equal.



[Observe that an equilateral triangle must be isosceles,

BOOK I. but that an isosceles triangle may or may not be equilateral.]

## XXVI.

A scalene triangle is that which has three unequal sides.

## XXVII.

A right angled triangle is that which has a right angle.

## XXVIII.

An obtuse angled triangle, is that which has an obtuse angle.



## XXIX.

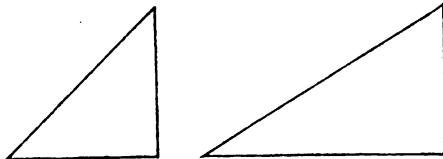
An acute angled triangle is that which has three acute angles.

[Observe that no triangle can have more than one right angle, nor more than one obtuse angle.]

Also a right angled triangle may be isosceles or not.

Fig. 1.

Fig. 2.

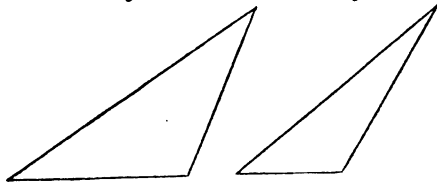


*Figs. 1 and 2.* are both right angled triangles. *Fig. 1.* is isosceles; *fig. 2.* is not isosceles.

Also an obtuse angled triangle may be isosceles or not.

Fig. 1.

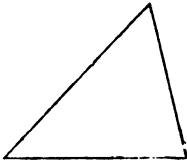
Fig. 2.



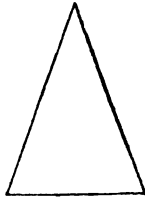
*Fig. 1.* is isosceles; *fig. 2.* is not isosceles.

Also an acute angled triangle may have all its three sides BOOK I  
unequal, in which case it is scalene; or two of them equal, in  
which case it is isosceles; or all three sides equal, in which  
case it is equilateral.

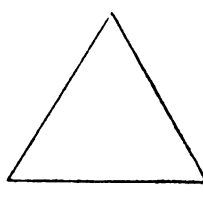
*Fig. 1.*



*Fig. 2.*



*Fig. 3.*



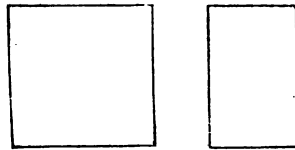
*Fig. 1.* is a scalene acute angled triangle.

*Fig. 2.* is an isosceles acute angled triangle.

*Fig. 3.* is an equilateral acute angled triangle.]

### XXX.

Of four-sided figures, a square is that which has all its sides equal, and all its angles right angles.

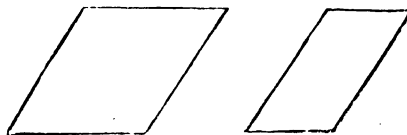


### XXXI.

An oblong is that which has all its angles right angles, but has not all its sides equal.

### XXXII.

A rhombus is that which has its sides equal, but its angles are not right angles.



## BOOK I.

## XXXIII.

A rhomboid is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.

## XXXIV.

All other four-sided figures besides these are called Trapeziums.

## XXXV.

Parallel straight lines are such as are in the same plane, and which, being produced ever so far both ways, do not meet.

Fig. 1.

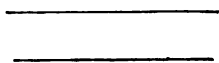
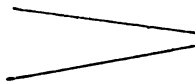


Fig. 2.



[Fig. 1. lines parallel; *fig. 2.* lines not parallel.

Observe particularly the words, *in the same plane.*]

## A.

A parallelogram is a four-sided figure, of which the opposite sides are parallel; and the diameter is the straight line joining two of its opposite angles.

Fig. 1.

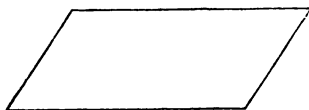


Fig. 2.

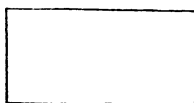
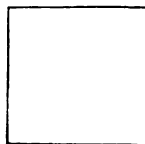


Fig. 3.



[All the above figures are parallelograms; those below are not parallelograms, both being four-sided figures. *Fig. 4.* has one pair of opposite sides parallel.

Fig. 4.

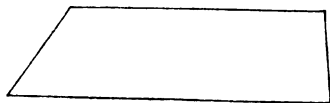


Fig. 5.



*Fig. 5.* has no pair of parallel sides.]

## POSTULATES.

## I.

LET it be granted that a straight line may be drawn from any one point to any other point.

## II.

That a terminated straight line may be produced to any length in a straight line.

## III.

And that a circle may be described from any centre, at any distance from that centre.

[These three postulates are equivalent in practice to allowing a pen, a ruler, and a pair of compasses. Having these, the reader will be able to draw any of the figures in this book.]

---

## AXIOMS.

## I.

THINGS which are equal to the same are equal to one another.

## II.

If equals be added to equals, the wholes are equal.

## III.

If equals be taken from equals, the remainders are equal.

## IV.

If equals be added to unequals, the wholes are unequal.

## V.

If equals be taken from unequals, the remainders are unequal.

## BOOK I

## VI.

Things which are double of the same, are equal to one another.

## VII.

Things which are halves of the same, are equal to one another.

## VIII.

Magnitudes which coincide with one another, that is, which exactly fill the same space, are equal to one another.

## IX.

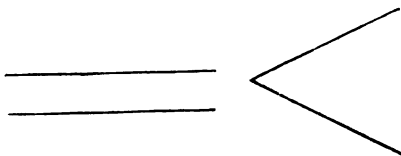
The whole is greater than its part.

## X.

Two straight lines cannot inclose a space.

Fig. 1.

Fig. 2.



[In *fig. 1*. no single additional straight line could be made to inclose a space; in *fig. 2*. a third straight line could be drawn across the other two so as to inclose a space.]

## XI.

All right angles are equal to one another.

## XII.

“If a straight line meets two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced, shall at length meet upon that side on which are the angles which are less than two right angles.”

[These are truths which no person can possibly doubt, who understands the terms in which they are expressed. There

are others besides these, but these are sufficient for the purpose. The person who admits these will find himself compelled to admit all that follows to the end of the First Book of Euclid. BOOK I.

If the reader finds any difficulty in understanding or admitting the truth of the last Axiom, he may omit all further consideration of it till he has read to the end of the 28th Proposition of the First Book.]

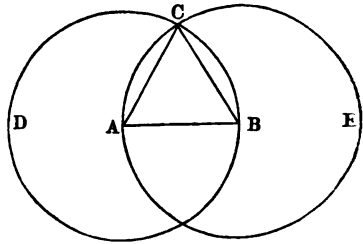
## BOOK I.

## PROPOSITION I. PROBLEM.

*To describe an equilateral triangle upon a given finite straight line.*

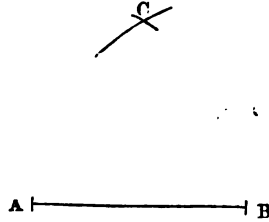
Let AB be the given straight line:  
it is required to describe an equilateral triangle upon it.

From the centre A, at the distance AB, describe<sup>a</sup> the circle BCD;  
and from the centre B, at the distance BA, describe the circle ACE;  
and from the point C, in which the circles cut one another, draw the straight lines<sup>b</sup>, CA, CB, to the points AB;  
ABC shall be an equilateral triangle.



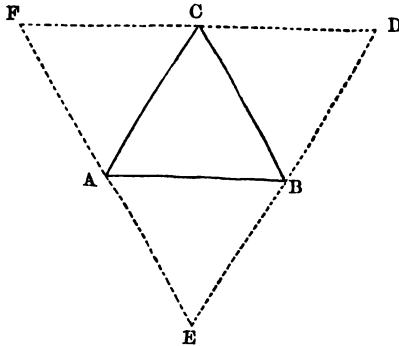
Because the point A is the centre of the circle BCD,  
AC is equal<sup>c</sup> to AB;  
and because the point B is the centre of the circle ACE,  
BC is equal to BA.  
But it has been proved that AC is equal to AB;  
therefore AC, BC are each of them equal to AB;  
but things which are equal to the same are equal to one another<sup>d</sup>  
therefore AC is equal to BC;  
wherefore AC, AB, BC are equal to one another;  
and the triangle ABC is therefore equilateral,  
and it is described upon the given straight line AB.  
Which was required to be done.

[There is no necessity in drawing this figure to complete the circles. Place one point of the compasses on A and the other point on B. Then with centre A and distance AB draw a small arc near where the two circles are likely to intersect. Then with centre B and distance BA touch the arc in the point C. Then with a pen and ruler join CA, CB.



Before proceeding further, practise drawing equilateral triangles as follows : —

Having made the triangle ABC, describe another equilateral triangle on the opposite side of AB, and also one on BC and on AC. The three triangles thus constructed can be made to lie exactly upon ABC. Cut the figure out in paper, and turn each of the triangles AEB, CDB, AFC on AB, BC, CA as hinges till the points D, E, F coincide. The figure thus formed is the first of the “five regular solids, and is called a tetrahedron.”]



PROP. II. PROB.

*From a given point to draw a straight line equal to a given straight line.*

Let A be the given point, and BC the given straight line; it is required to draw from the point A a straight line equal to BC.

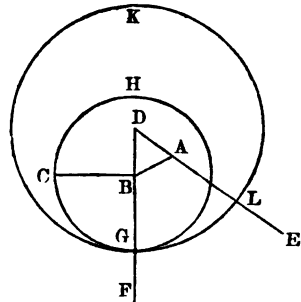
BOOK I From the point A to B draw <sup>a</sup> the straight line AB ;

<sup>a</sup> 1 Post. and upon it describe <sup>b</sup> the equilateral triangle DAB,

<sup>b</sup> 1. 1. and produce<sup>c</sup> the straight lines DA, DB, to E and F ;

<sup>c</sup> 2 Post. from the centre B, at the distance BC, describe<sup>d</sup> the circle CGH ;

<sup>d</sup> 3 Post. and from the centre D, at the distance DG, describe the circle GKL.



AL shall be equal to BC.

Because the point B is the centre of the circle CGH,

<sup>e</sup> 15 Def. BC is equal<sup>e</sup> to BG ;

and because D is the centre of the circle GKL,

DL is equal to DG,

and DA, DB, parts of them, are equal ;

<sup>f</sup> 3 Ax. therefore the remainder AL is equal to the remainder<sup>f</sup> BG.

But it has been shown that BC is equal to BG ;

wherefore AL and BC are each of them equal to BG ;

and things that are equal to the same are equal to one another<sup>h</sup> ;

<sup>h</sup> 1 Ax.

therefore the straight line AL is equal to BC.

Wherefore, from the given point A,

a straight line AL has been drawn equal to the given straight line BC.

Which was to be done.

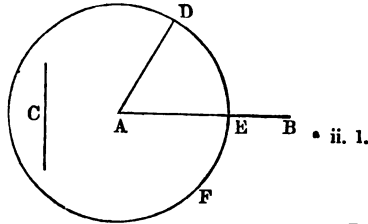
[For the future, when it is required to draw a right line from a given point A equal to a given right line, BC, place the two points of your compasses on B and C and remove one of them to A, and the other to L, and with a pen and ruler draw the straight line AL.]

PROP. III. PROB.

*From the greater of two given straight lines to cut off a part equal to the less.*

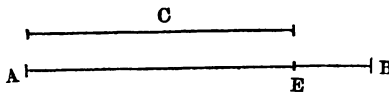
Let AB and C be the two given straight lines,  
whereof AB is the greater.  
It is required to cut off from AB,  
the greater, a part equal to  
C, the less.

From the point A draw<sup>a</sup> the  
straight line AD equal to C,  
and from the centre A, and at the  
distance AD, describe<sup>b</sup> the circle  
DEF;



And because A is the centre of the circle DEF,  
AE shall be equal to AD;  
but the straight line C is likewise equal to AD;  
whence AE and C are each of them equal to AD;  
wherefore the straight line AE is equal to C<sup>c</sup>,  
and from AB, the greater of two straight lines,  
a part AE has been cut off equal to C the less.  
Which was to be done.

[There is no necessity to draw this figure whenever it is  
required to cut off a part from the greater of two straight  
lines equal to the less.  
Place the points of the  
compasses at the extrem-  
ities of the less right line,  
and then keeping them at  
the same distances from each other, place them respectively  
on A and E.]

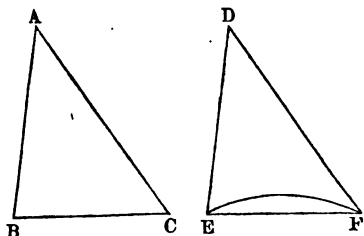


## BOOK I

## PROP. IV. THEOREM.

*If two triangles have two sides of the one equal to two sides of the other, each to each; and have likewise the angles contained by those sides equal to one another; they shall likewise have their bases, or third sides, equal; and the two triangles shall be equal; and their other angles shall be equal, each to each, viz. those to which the equal sides are opposite.*

Let ABC, DEF be two triangles,  
 which have the two sides AB, AC equal to the two sides  
 DE, DF,  
 each to each, viz. AB to DE, and AC to DF;  
 and the angle BAC equal to  
 the angle EDF,  
 the base BC shall be equal  
 to the base EF;  
 and the triangle ABC to the  
 triangle DEF;  
 and the other angles of  
 which the equal sides  
 are opposite,  
 shall be equal each to each,  
 viz. the angle ABC to the angle DEF,  
 and the angle ACB to DFE.




---

For, if the triangle ABC be applied to DEF,  
 so that the point A may be on D,  
 and the straight line AB upon DE;  
 the point B shall coincide with the point E,  
 because AB is equal to DE;  
 and AB coinciding with DE,  
 AC shall coincide with DF,  
 because the angle BAC is equal to the angle EDF;  
 wherefore also the point C shall coincide with the point F,  
 because the straight line AC is equal to DF:

But the point B coincides with the point E ;  
 wherefore the base BC shall coincide with the base EF,  
 because the point B coinciding with E, and C with F,  
 if the base BC does not coincide with the base EF,  
 two straight lines would inclose a space,  
 which is impossible.\*

\* 10 Ax.

Therefore the base BC shall coincide with the base EF,  
 and be equal to it.

Wherefore the whole triangle ABC shall coincide  
 with the whole triangle DEF,  
 and be equal to it ;

and the other angles of the one shall coincide  
 with the remaining angles of the other,  
 and be equal to them,  
 viz. the angle ABC to the angle DEF,  
 and the angle ACB to DFE.

Therefore, if two triangles  
 have two sides of the one equal to two sides of the other,  
 each to each,  
 and have likewise the angles contained by those sides equal  
 to one another,  
 their bases shall likewise be equal,  
 and the triangles be equal,  
 and their other angles to which the equal sides are opposite  
 shall be equal, each to each.  
 Which was to be demonstrated.

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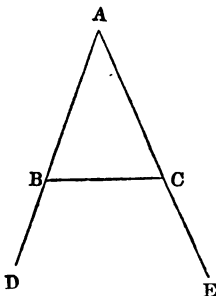
[Cut a triangle of any kind, BAC, out of a piece of paper  
 or cardboard, and, laying it on a larger piece, draw DE and  
 DF so as exactly to coincide with AB and AC. Then,  
 removing the triangle, join the points E and F by the use of  
 a pen and straight ruler. It will be seen that the points E  
 and F coinciding with B and C, the whole line BC will  
 exactly coincide with EF, unless it be possible for two  
 straight lines to enclose a space.]

## BOOK I.

## PROP. V. THEOR.

*The angles at the base of an Isosceles triangle are equal to one another ; and if the equal sides be produced, the angles upon the other side of the base shall be equal.*

Let ABC be an Isosceles triangle, of which the side AB is equal to AC, and let the straight lines AB, AC be produced to D and E, the angle ABC shall be equal to the angle ACB, and the angle CBD to the angle BCE.



In BD take any point F, and from AE the greater cut off AG equal<sup>a</sup> to AF the less, and join FC, GB.

<sup>a</sup> iii. 1.

Because AF is equal to AG, and AB to AC, the two sides FA, AC are equal to the two GA, AB, each to each ; and they contain the angle FAG common to the two triangles AFC, AGB ; therefore the base FC is equal<sup>b</sup> to the base GB ; and the triangle AFC to the triangle AGB ;

<sup>b</sup> iv. 1.

and the remaining angles of the one are equal<sup>b</sup> to the remaining angles of the other,

each to each, to which the equal sides are opposite ;

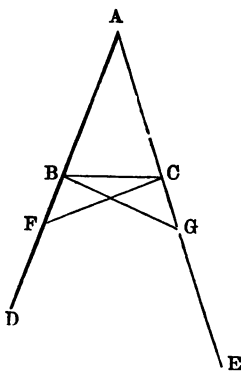
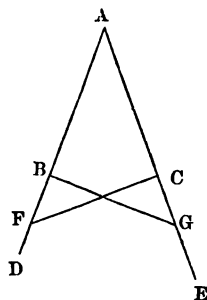
viz. the angle ACF to the angle ABG, and the angle AFC to the angle AGB :

And because the whole AF is equal to the whole AG,

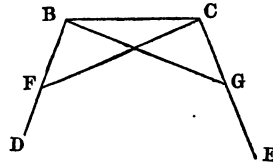
of which the parts AB, AC are equal ;

<sup>c</sup> 3 Ax.

the remainder BF shall be equal<sup>c</sup> to the remainder CG ;



and FC was proved to be equal to GB;  
therefore the two sides BF, FC  
are equal to the two CG, GB, each to each;  
and the angle BFC is equal to the angle CGB,  
and the base BC is common to the  
two triangles BFC, CGB;  
wherefore the triangles are equal<sup>b</sup>  
and their remaining angles, each  
to each,  
to which the equal sides are opposite;  
therefore the angle FBC is equal to the angle GCB,



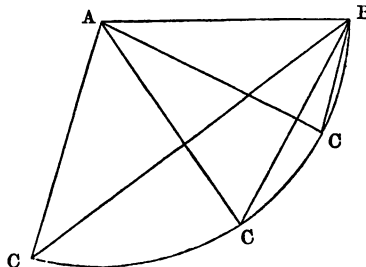
<sup>b</sup> iv. 1.

and the angle BCF to the angle CBG.

And, since it has been demonstrated,  
that the whole angle ABG is equal to the whole ACF,  
the parts of which, the angles CBG, BCF, are also equal,  
the remaining angle ABC is therefore equal  
to the remaining angle ACB,  
which are the angles at the base of the triangle ABC.  
And it has also been proved  
that the angle FBC is equal to the angle GCB,  
which are the angles upon the other side of the base.  
Therefore the angles at the base, &c. Q. E. D.

**COROLLARY.** Hence every equilateral triangle is also  
equiangular.

[The method of drawing isosceles triangles is as follows.  
Draw a right line AB. With  
one point of the compasses  
on A, and with the distance  
AB describe the arc of a  
circle less than a semicircle.  
Upon it take any points,  
C, C, C. Join AC. Then  
if BC be joined, the triangle  
BAC will be isosceles, hav-  
ing the sides BA, AC equal to each other.]



## BOOK I.

## PROP. VI. THEOR.

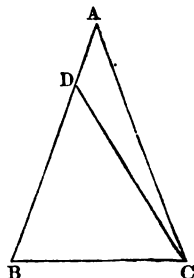
*If two angles of a triangle be equal to one another, the sides also which subtend, or are opposite to, the equal angles, shall be equal to one another.*

Let ABC be a triangle  
having the angle ABC equal to the angle ACB;  
the side AB is also equal to the side AC.

For, if AB be not equal to AC,  
one of them is greater than the other :

Let AB be the greater;

<sup>a</sup> iii. 1. and from it cut<sup>a</sup> off DB equal to AC, the  
less,  
and join DC;



Therefore, because in the triangles DBC, ACB,  
DB is equal to AC,  
and BC common to both,  
the two sides, DB, BC are equal to the two AC, CB, each  
to each;

and the angle DBC is equal to the angle ACB;

therefore the base DC is equal to the base AB,

<sup>b</sup> iv. 1. and the triangle DBC is equal to the triangle<sup>b</sup> ACB,  
the less to the greater;  
which is absurd.

Therefore AB is not unequal to AC;

that is, it is equal to it.

Wherefore, if two angles, &c. Q. E. D.

COR. Hence every equiangular triangle is also equilateral.

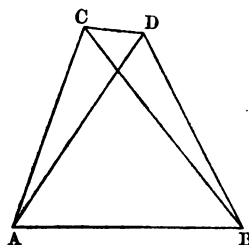
[To exhibit the truth of this, draw first the base BC, and place a triangular piece of cardboard with one of its edges coincident with BC, the angular point coinciding with B, and the other edge inclined at any angle, as CBA. Draw BA. Then turn it round, and make the same angular point coincide with C, the edge as before coinciding with BC, and make the angle BCA. The sides BA and CA being produced till they meet will be equal.]

PROP. VII. THEOR.

*Upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base equal to one another, and likewise those which are terminated in the other extremity.*

If it be possible, let there be two triangles  $ACB$ ,  $ADB$ , upon the same base  $AB$ , and upon the same side of it, which have their sides  $CA$ ,  $DA$ , terminated in the extremity  $A$  of the base equal to one another, and likewise their sides  $CB$ ,  $DB$ , that are terminated in  $B$ .

Join  $CD$ .



CASE I. Then, in the case in which the vertex of each of the triangles is without the other triangle,

because  $AC$  is equal to  $AD$ ,

the angle  $ACD$  is equal<sup>a</sup> to the angle  $ADC$ .

<sup>a</sup> v. 1.

But the angle  $ACD$  is greater than the angle  $BCD$ ;

therefore the angle  $ADC$  is greater also than  $BCD$ ;

much more then is the angle  $BDC$  greater than the angle  $BCD$ .

Again, because  $CB$  is equal to  $DB$ ,

the angle  $BDC$  is equal<sup>a</sup> to the angle  $BCD$ ;

but it has been demonstrated to be greater than it;

which is impossible.

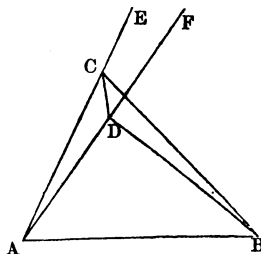
CASE II. But if one of the vertices, as  $D$ , be within the other triangle  $ACB$ ;

produce  $AC$ ,  $AD$  to  $E$ ,  $F$ ;

therefore, because  $AC$  is equal to  $AD$ ;

in the triangle  $ACD$ ,

the angles  $ECD$ ,  $FDC$  upon the other side of the base  $CD$  are equal<sup>a</sup> to one another,



BOOK I. but the angle  $ECD$  is greater than the angle  $BCD$  ;  
 wherefore the angle  $FDC$  is likewise greater than  $BCD$  ;  
 much more then is the angle  $BDC$  greater than the angle  $BCD$  ;  
 Again, because  $CB$  is equal to  $DB$ ,  
 \* v. 1. the angle  $BDC$  is equal \* to the angle  $BCD$  ;  
 but  $BDC$  has been proved to be greater than the same  $BCD$  ;  
 which is impossible.

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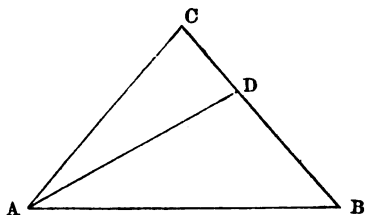
The case in which the vertex of one triangle is upon a side of the other, needs no demonstration.

Therefore, upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base equal to one another, and likewise those which are terminated in the other extremity. Q. E. D.

---

[Observe it is possible that  $CA$  may be equal to  $DA$ , or  $CB$  equal to  $DB$ . It is not possible that both  $CA$  can be equal to  $DA$ , and  $CB$  equal to  $DB$ .

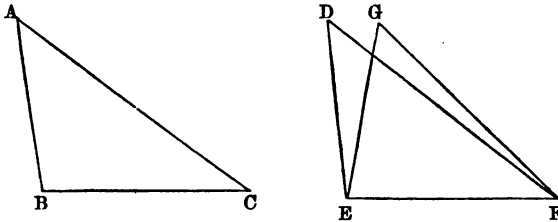
The case in which the vertex of one triangle is upon a side of the other, is shown in the annexed figure; where it is manifest that, whether  $CA$  be equal to  $DA$  or not,  $CB$  cannot be equal to  $DB$ .]



## PROP. VIII. THEOR.

*If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal; the angle which is contained by the two sides of the one shall be equal to the angle contained by the two sides equal to them, of the other.*

Let ABC, DEF be two triangles,  
having the two sides AB, AC,  
equal to the two sides DE, DF,



each to each, viz. AB to DE,  
and AC to DF;  
and also the base BC equal to the base EF.  
The angle BAC is equal to the angle EDF.

---

For, if the triangle ABC be applied to DEF,  
so that the point B be on E,  
and the straight line BC upon EF;  
the point C shall also coincide with the point F,  
because BC is equal to EF;  
therefore BC coinciding with EF,  
BA and AC shall coincide with ED and DF;  
for, if the base BC coincides with the base EF,  
but the sides BA, CA do not coincide with the sides  
ED, FD,  
but have a different situation as EG, FG;  
then, upon the same base EF, and upon the same side of it,

BOOK I there can be two triangles that have their sides which are terminated in one extremity of the base equal to one another, and likewise their sides terminated in the other extremity :

<sup>a</sup> vii. 1. But this is impossible <sup>a</sup>;  
therefore, if the base BC coincides with the base EF,  
the sides BA, AC cannot but coincide with the sides ED, DF;  
wherefore likewise the angle BAC coincides with the angle  
EDF,

<sup>b</sup> 8 Ax. and is equal <sup>b</sup> to it.  
Therefore, if two triangles, &c. Q. E. D.

[Compare this with Prop. IV., and it will appear that the two triangles are equal in all respects.

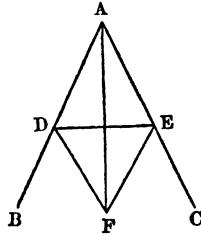
What is proved in this Proposition is the converse of what was proved in the fourth.]

## PROP. IX. PROB.

*To bisect a given rectilinear angle, that is, to divide it into two equal angles.*

Let BAC be the given rectilinear angle,  
it is required to bisect it.

Take any point D in AB,  
and from AC cut <sup>a</sup> off AE equal to AD;  
join DE,  
and upon it describe <sup>b</sup> an equilateral  
triangle DEF;  
then join AF;  
the straight line AF bisects the angle BAC.



<sup>a</sup> iii. 1.

<sup>b</sup> i. 1.

Because AD is equal to AE,  
and AF is common to the two triangles DAF, EAF;  
the two sides DA AF, are equal to the two sides EA, AF,  
each to each;  
and the base DF is equal to the base EF;  
therefore the angle DAF is equal <sup>c</sup> to the angle EAF: <sup>d</sup> viii. 1.  
wherefore the given rectilinear angle BAC is bisected by the  
straight line AF.  
Which was to be done.

[It is not necessary that the triangle DEF be equilateral.  
It is sufficient for the purpose if it be isosceles, having DF  
equal to FE.]

To draw it, place one point of the compasses on D, and  
taking care that the opening shall be such as shall place the  
other point at a distance from it greater than half the line  
DE, draw a small arc at F. Then with the point placed on  
E with the same opening, touch the small arc in F. AF  
will be the line which bisects the angle.]

## BOOK I

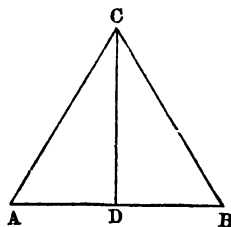
## PROP. X. PROB.

*To bisect a given finite straight line, that is, to divide it into two equal parts.*

Let AB be the given straight line;  
it is required to divide it into two equal parts.

- i. 1. Describe <sup>a</sup> upon it an equilateral triangle ABC,  
• ix. 1. and bisect <sup>b</sup> the angle ACB by the straight line CD.  
AB is cut into two equal parts in the point D.

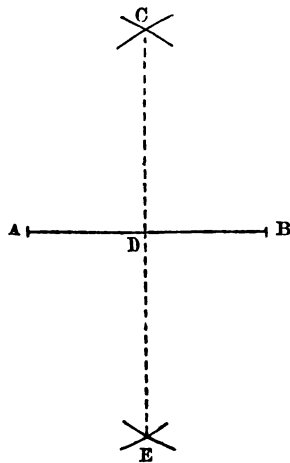
Because AC is equal to CB,  
and CD common to the two triangles  
ACD, BCD;  
the two sides AC, CD are equal to  
BC, CD, each to each;  
and the angle ACD is equal to the  
angle BCD;



- iv. 1. therefore the base AD is equal to the base <sup>c</sup> DB,  
and the straight line AB is divided into  
two equal parts in the point D.  
Which was to be done.

[The most expeditious mode of  
bisecting a straight line is as fol-  
lows:—

Place the point of the compasses  
on A, and with an opening larger  
than half the line, draw a small arc  
at C above, and another at E be-  
low the line. Then with the point  
at B and the same aperture touch  
the arcs in the point C and E.  
Then with a ruler placed upon C  
and E, observe where it cuts the  
line AB, and mark the point D.]

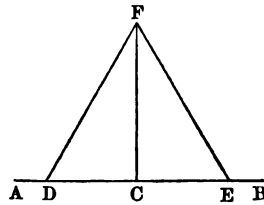


PROP. XI. PROB.

*To draw a straight line at right angles to a given straight line, from a given point in the same.*

Let AB be a given straight line,  
and C a point given in it;  
it is required to draw a straight line from the point C at  
right angles to AB.

Take any point D in AC,  
and <sup>a</sup> make CE equal to CD,  
and upon DE describe <sup>b</sup> the equi-  
lateral triangle DFE,  
and join FC,



<sup>a</sup> iii. 1.

<sup>b</sup> i. 1.

the straight line FC drawn from the given point C is at right  
angles to the given straight line AB.

Because DC is equal to CE,  
and FC common to the two triangles DCF, ECF;  
the two sides DC, CF are equal to the two EC, CF, each  
to each;  
and the base DF is equal to the base EF;  
therefore the angle DCF is equal <sup>c</sup> to the angle ECF;  
and they are adjacent angles.

<sup>c</sup> viii. 1.

But, when the adjacent angles which one straight line makes  
with another straight line are equal to one another,  
each of them is called a right <sup>d</sup> angle;  
therefore each of the angles DCF, ECF is a right angle.  
Wherefore, from the given point C, in the given straight  
line AB,

<sup>d</sup> 10 Def.

FC has been drawn at right angles to AB.  
Which was to be done.

BOOK I. [Here again it is not necessary that the triangle DFE be equilateral. It is sufficient if it be isosceles, having DF equal to FE.]

**COR.** By help of this problem it may be demonstrated, that two straight lines cannot have a common segment.

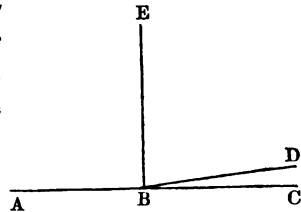
If it be possible, let the two straight lines ABC, ABD have the segment AB common to both of them.

From the point B draw BE at right angles to AB; and because ABC is a straight line, the angle CBE is equal<sup>a</sup> to the angle EBA;

in the same manner, because ABD is a straight line,

the angle DBE is equal to the angle EBA, wherefore the angle DBE is equal to the angle CBE, the less to the greater; which is impossible;

therefore two straight lines cannot have a common segment.



<sup>a</sup> 10 Def. 1. the angle CBE is equal<sup>a</sup> to the angle EBA;

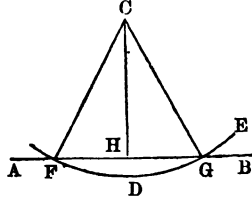
## PROP. XII. PROB.

## BOOK I.

*To draw a straight line perpendicular to a given straight line of an unlimited length, from a given point without it.*

Let AB be the given straight line,  
which may be produced to any length both ways,  
and let C be a point without it.

It is required to draw a straight line  
perpendicular to AB from the  
point C.



Take any point D upon the other  
side of AB,

and from the centre C, at the distance CD,  
describe <sup>a</sup> the circle EGF meeting AB in FG ;

and bisect <sup>b</sup> FG in H,

and join CF, CH, CG ;

the straight line CH, drawn from the given point C,  
is perpendicular to the given straight line AB.

<sup>a</sup> 3 Post.

<sup>b</sup> x. 1.

Because FH is equal to HG,

and HC common to the two triangles FHC, GHC,

the two sides FH, HC are equal to the two GH, HC, each  
to each ;

and the base CF is equal <sup>c</sup> to the base CG ;

therefore the angle CHF is equal <sup>d</sup> to the angle CHG ;

<sup>c</sup> 15 Def. 1.

<sup>d</sup> viii. 1.

and they are adjacent angles ; but when a straight line stand-  
ing on a straight line makes the adjacent angles equal to  
one another, each of them is a right angle ;

and the straight line which stands upon the other is called  
a perpendicular to it ;

therefore from the given point C a perpendicular CH has  
been drawn to the given straight line AB.

Which was to be done.

[Instead of describing the arc of a circle, touch the points  
F and G with the points of the compasses.]

## BOOK I.

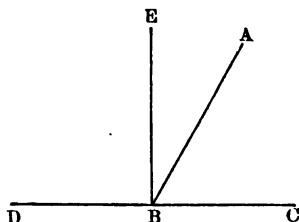
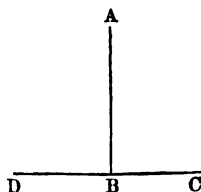
## PROP. XIII. THEOR.

*The angles which one straight line makes with another upon the one side of it, are either two right angles, or are together equal to two right angles.*

Let the straight line AB make with CD, upon one side of it, the angles CBA, ABD: these are either two right angles, or are together equal to two right angles.

For if the angle CBA be equal to ABD,

• Def. 10. each of them is a right<sup>a</sup> angle;



but, if not, from the point B

• xi. 1.

draw BE at right angles<sup>b</sup> to CD;

therefore the angles CBE, EBD are two right angles<sup>a</sup>;

and because CBE is equal to

the two angles CBA, ABE together,

add the angle EBD to each of these equals;

• 2 Ax.

therefore the angle CBE, EBD are equal<sup>c</sup> to

the three angles CBA, ABE, EBD.

Again, because the angle DBA is equal to

the two angles DBE, EBA,

add to these equals the angle ABC,

therefore the angles DBA, ABC are equal to

the three angles DBE, EBA, ABC,

but the angles CBE, EBD have been demonstrated

to be equal to the same three angles;

• 1 Ax.

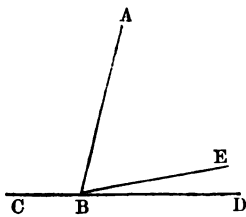
and things that are equal to the same are equal<sup>d</sup> to one another;

therefore the angles CBE, EBD are equal to the angles DBA, ABC;  
 but CBE, EBD are two right angles;  
 therefore DBA, ABC are together equal to two right angles.  
 Wherefore, when a straight line, &c. Q. E. D.

## PROP. XIV. THEOR.

*If, at a point in a straight line, two other straight lines, upon the opposite sides of it, make the adjacent angles together equal to two right angles, these two straight lines shall be in one and the same straight line.*

At the point B in the straight line AB,  
 let the two straight lines BC, BD, upon the opposite sides of AB, make the adjacent angles ABC, ABD equal together to two right angles.  
 BD is in the same straight line with CB.



For, if BD be not in the same straight line with CB, let BE be in the same straight line with it;

Therefore, because the straight line AB makes angles with the straight line CBE, upon one side of it, the angles ABC, ABE are together equal <sup>a</sup> to two right angles;  
 but the angles ABC, ABD are likewise together equal to two right angles;  
 therefore the angles CBA, ABE are equal to the angles CBA, ABD.

<sup>a</sup> xiii. 1.

Take away the common angle ABC, the remaining angle ABE is equal <sup>b</sup> to the remaining angle ABD, the less to the greater, which is impossible;  
 therefore BE is not in the same straight line with BC.  
 And, in like manner, it may be demonstrated,

<sup>b</sup> 3 Ax.

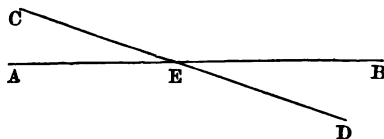
**BOOK I** that no other can be in the same straight line with it but BD, which therefore is in the same straight line with CB. Wherefore, if at a point, &c. Q. E. D.

PROP. XV. THEOR.

*If two straight lines cut one another, the vertical, or opposite, angles shall be equal.*

Let the two straight lines AB, CD cut one another in the point E;  
the angle AEC shall be equal to the angle DEB,  
and CEB to AED.

Because the straight  
line AE  
makes with CD the angles  
CEA, AED,



these angles are together

<sup>a</sup> equal <sup>a</sup> to two right angles.

Again, because the straight line DE makes with AB,  
the angles AED, DEB,

these also are together equal <sup>a</sup> to two right angles;  
and CEA, AED have been demonstrated to be equal  
to two right angles;

wherefore the angles CEA, AED are equal to  
the angles AED, DEB.

Take away the common angle AED,

<sup>b</sup> 3 Ax. and the remaining angle CEA is equal <sup>b</sup> to  
the remaining angle DEB.

In the same manner it can be demonstrated,  
that the angles CEB, AED are equal.

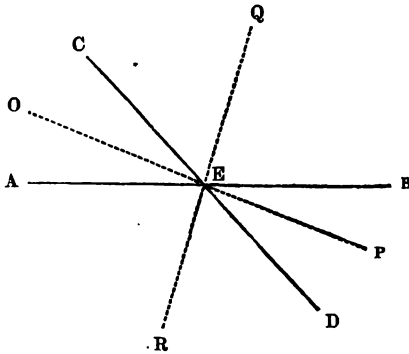
Therefore, if two straight lines, &c. Q. E. D.

COR. 1. From this it is manifest, that,

If two straight lines cut one another, the angles they make at  
the point where they cut, are together equal to four  
right angles.

[For CEA and CEB are together equal to two right angles, and AED and DEB are together equal to two right angles, or all the four angles are equal to four right angles.] BOOK I.

COR. 2. And consequently that  
All the angles made by any number of lines meeting in one point, are together equal to four right angles.



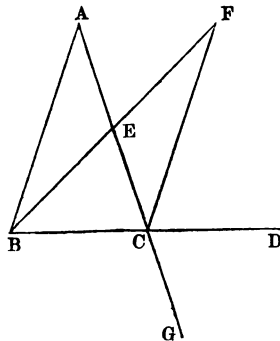
PROP. XVI. THEOR.

*If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.*

Let ABC be a triangle,  
and let its side BC be produced to D,  
the exterior angle ACD is greater than either of the interior  
opposite angles CBA, BAC.

Bisect <sup>a</sup> AC in E,  
join BE and produce it to F,  
and make EF equal to BE;  
join also FC.

Because AE is equal to EC,  
and BE to EF;  
AE, EB are equal to CE, EF,  
each to each;



<sup>a</sup> x. 1.

**BOOK I** and the angle AEB is equal <sup>b</sup> to the angle CEF,

<sup>a</sup> xv. 1. because they are opposite vertical angles ;

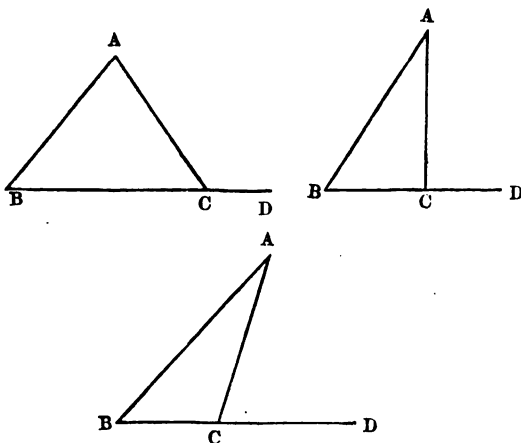
<sup>c</sup> iv. 1. therefore the base AB is equal <sup>c</sup> to the base CF,  
and the triangle AEB to the triangle CEF,  
and the remaining angles to the remaining angles,  
each to each, to which the equal sides are opposite ;  
wherefore the angle BAE is equal to the angle ECF ;  
but the angle ECD is greater than the angle ECF ;  
therefore the angle ACD is greater than BAE :

In the same manner, if the side BC be bisected, and AC pro-  
duced to G, it may be demonstrated that the angle BCG,

<sup>d</sup> xv. 1. which is equal <sup>d</sup> to the angle ACD,  
is greater than the angle ABC.

Therefore, if one side, &c. Q. E. D.

[Observe that the angle ACD *has been* proved greater than ~~ABC~~ <sup>ABC</sup>, and that the angle BCG *may be* proved greater than ~~ABC~~ <sup>ABC</sup>; and as ACD and BCG are equal, it follows that either of them is greater than either of the interior opposite angles ABC, BAC. The exterior angle may be greater than, equal to, or less than the interior angle which is adjacent to it, as may be seen in the annexed figures.]



PROP. XVII. THEOR.

*Any two angles of a triangle are together less than two right angles.*

Let ABC be any triangle ;  
any two of its angles together are less  
than two right angles.

Produce BC to D ;

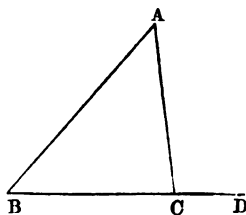
and because ACD is the exterior angle  
of the triangle ABC,

ACD is greater<sup>a</sup> than the interior and  
opposite angle ABC ;

to each of these add the angle ACB ;  
therefore the angles ACD, ACB are greater than  
the angles ABC, ACB ;

but ACD, ACB are together equal<sup>b</sup> to two right angles ;  
therefore the angles ABC, BCA are less than two right  
angles.

In like manner, it may be demonstrated that BAC, ACB,  
as also CAB, ABC, are less than two right angles.  
Therefore any two angles, &c. Q. E. D.



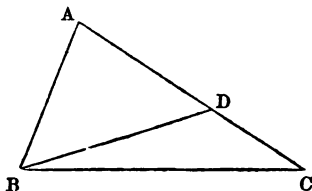
<sup>a</sup> xvi. 1.

<sup>b</sup> xiii. 1.

PROP. XVIII. THEOR.

*The greater side of every triangle is opposite to the greater angle.*

Let ABC be a triangle,  
of which the side AC is greater  
than the side AB ;  
the angle ABC is also greater  
than the angle BCA.



Because AC is greater than AB,  
make<sup>a</sup> AD equal to AB,  
and join BD ;

<sup>a</sup> iii. 1.

**BOOK I.** and because  $\angle ADB$  is the exterior angle of the triangle  $BDC$ ,  
 it is greater <sup>b</sup> than the interior and opposite angle  $DCB$ ;  
 but  $\angle ADB$  is equal <sup>a</sup> to  $\angle ABD$ ,  
 because the side  $AB$  is equal to the side  $AD$ ;  
 therefore the angle  $ABD$  is likewise greater than the angle  
 $ACB$ .

Wherefore much more is the angle  $ABC$  greater than  $ACB$ .  
 Therefore the greater side, &c. Q. E. D.

PROP. XIX. THEOR.

*The greater angle of every triangle is subtended by the greater side, or has the greater side opposite to it.*

Let  $ABC$  be a triangle,  
 of which the angle  $ABC$  is greater than the angle  $BCA$ ;  
 the side  $AC$  is likewise greater than the side  $AB$ .

For, if it be not greater,  
 $AC$  must either be equal to  
 $AB$ ,

or less than it;

it is not equal, because then  
 the angle  $ABC$  would  
 be equal <sup>a</sup> to the angle  
 $ACB$ ;

but it is not;

therefore  $AC$  is not equal to  $AB$ ;

neither is it less; because then

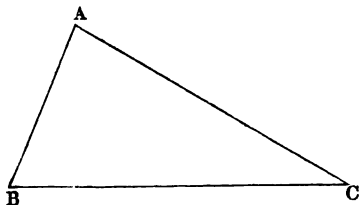
the angle  $ABC$  would be less <sup>b</sup> than the angle  $ACB$ ;  
 but it is not;

therefore the side  $AC$  is not less than  $AB$ ;

and it has been shown that it is not equal to  $AB$ ;

therefore  $AC$  is greater than  $AB$ .

Wherefore the greater angle, &c. Q. E. D.



[The latter of these propositions is the converse of the former: the former shows that if one side be greater than another, the angle opposite that side is greater than the angle opposite the other; the latter, that if one angle is greater than another, the side opposite that angle is greater than the side opposite the other.]

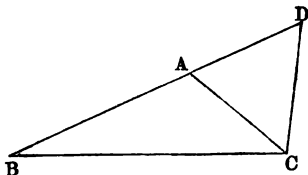
BOOK I

PROP. XX. THEOR.

*Any two sides of a triangle are together greater than the third side.*

Let ABC be a triangle;  
any two sides of it together are greater than the third side,  
viz. the sides BA, AC greater than the side BC;  
and AB, BC greater than AC;  
and BC, CA greater than AB.

Produce BA to the point D,  
and make <sup>a</sup> AD equal to AC;  
and join DC.



<sup>a</sup> iii. 1.

Because DA is equal to AC,  
the angle ADC is likewise equal <sup>b</sup> to ACD;  
but the angle BCD is greater than the angle ACD;  
therefore the angle BCD is greater than the angle ADC;  
and because the angle BCD of the triangle DCB  
is greater than its angle BDC,  
and that the greater <sup>c</sup> side is opposite to the greater angle;  
therefore the side DB is greater than the side BC;  
but DB is equal to BA and AC;  
therefore the sides BA, AC are greater than BC.  
In the same manner it may be demonstrated,  
that the sides AB, BC are greater than CA,  
and BC, CA greater than AB.  
Therefore any two sides, &c. Q. E. D.

<sup>b</sup> v. 1.

<sup>c</sup> xix. 1.

## BOOK I

## PROP. XXI. THEOR.

*If from the ends of the side of a triangle there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle, but shall contain a greater angle.*

Let the two straight lines BD, CD be drawn from B, C, the ends of the side BC of the triangle ABC, to the point D within it;  
BD and DC are less than the other two sides BA, AC of the triangle,  
but contain an angle BDC greater than the angle BAC.

Produce BD to E; \_\_\_\_\_

And because two sides of a triangle are greater than the third side,

the two sides BA, AE of the triangle ABE are greater than BE.

To each of these add EC;  
therefore the sides BA, AC are greater than BE, EC.

Again, because the two sides CE, ED,

of the triangle CED, are greater than CD,

add DB to each of these;

therefore the sides CE, EB are greater than CD, DB;

but it has been shown that BA, AC

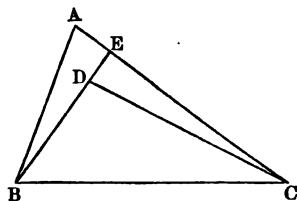
are greater than BE, EC,

much more then are BA, AC greater than BD, DC.

Again, because the exterior angle of a triangle is greater than the interior and opposite angle, the exterior angle BDC of the triangle CDE is greater than CED;

for the same reason,

the exterior angle CEB of the triangle ABE is greater than BAC;



and it has been demonstrated that the angle BDC  
is greater than the angle CEB ;  
much more then is the angle BDC  
greater than the angle BAC.  
Therefore, if from the ends of, &c. Q. E. D.

PROP. XXII. PROB.

*To make a triangle of which the sides shall be equal to three  
given straight lines, but any two whatever of these must be  
greater than the third.\**

\* xx. 1.

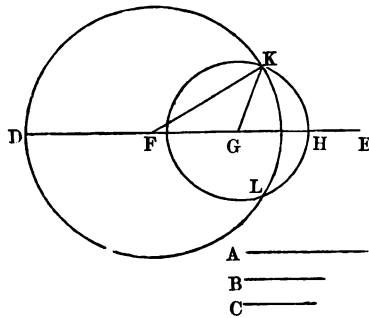
Let A, B, C be the three given straight lines,  
of which any two whatever are greater than the third, viz.

A and B greater than C ;  
A and C greater than B ;  
and B and C than A.

It is required to make a triangle,  
of which the *sides* shall be equal to A, B, C, each to each.

Take a straight line DE terminated at the point D,  
but unlimited towards E,  
and make<sup>a</sup> DF equal to A,  
FG to B,

and GH equal to C ;  
and from the centre F,  
at the distance FD,  
describe<sup>b</sup> the circle DKL ;  
and from the centre G,  
at the distance GH,  
describe<sup>b</sup> another circle HLK ;  
and join KF, KG ;  
the triangle KFG has its sides equal to the three straight  
lines A, B, C.



\* iii. 1.

<sup>b</sup> 3 Post.

Because the point F is the centre of the circle DKL,  
FD is equal<sup>c</sup> to FK ;  
but FD is equal to the straight line A ;

\* 15 Def.

BOOK I therefore FK is equal to A :

Again, because G is the centre of the circle LKH,

• 15 Def. GH is equal  $\circ$  to GK ;

but GH is equal to C ;

therefore also GK is equal to C ;

and FG is equal to B ;

therefore the three straight lines KF, FG, GK,

are equal to the three A, B, C :

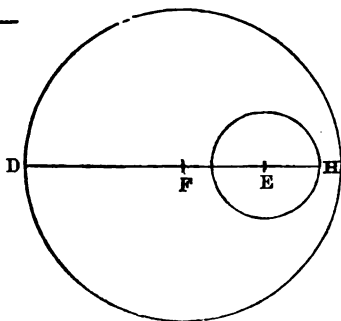
And therefore the triangle KFG has its three sides

KF, FG, GK

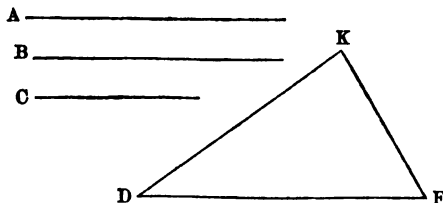
equal to the three given straight lines, A, B, C.

Which was to be done.

[Take three straight lines which do not fulfil the conditions required, that any A \_\_\_\_\_  
two should be greater B \_\_\_\_\_  
than the third, thus: C \_\_\_\_\_  
and the circles would not intersect at all, and no triangle such as is required could be drawn, as might have been expected from Prop. XX.



Another mode of drawing such a triangle would be as follows: Draw any line DF equal to A. From D as centre, with distance equal to B, describe a circular arc K; and from F as centre, with distance equal to C, draw a circular arc cutting the other in K. Join DK and KF. If B and C should be together equal to A, the point K would lie on the line DF. If they should together be less than A, the circles would have no point in common.]



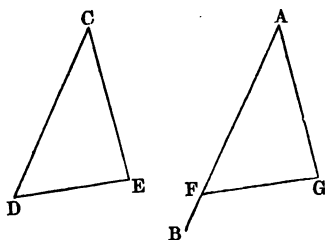
Join DK and KF. If B and C should be together equal to A, the point K would lie on the line DF. If they should together be less than A, the circles would have no point in common.]

PROP. XXIII. PROB.

*At a given point in a given straight line, to make a rectilineal angle equal to a given rectilineal angle.*

Let AB be the given straight line,  
and A the given point in it,  
and DCE the given rectilineal angle ;

it is required to make an angle at the given point A in the given straight line AB, that shall be equal to the given rectilineal angle DCE.



Take in CD, CE any points D, E,  
and join DE ;  
and make <sup>a</sup> the triangle AFG,  
the sides of which shall be equal to the three straight lines  
CD, DE, EC,  
so that CD be equal to AF,  
CE to AG,  
and DE to FG ;

<sup>a</sup> xxii. 1.

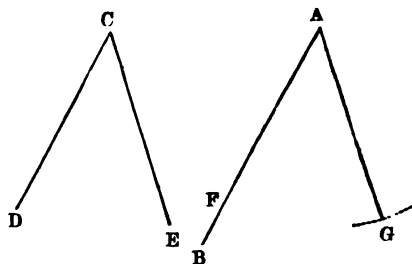
And because DC, CE are equal to FA, AG, each to each,  
and the base DE to the base FG ;  
the angle DCE is equal <sup>b</sup> to the angle FAG.  
Therefore, at the given point A in the given straight line AB,  
the angle FAG is made equal to the given rectilineal angle  
DCE.

<sup>b</sup> viii. 1.

Which was to be done.

[The best mode of drawing the angle required is by making CD and CE equal to each other. Then in AB,

BOOK I. cut off  $AF = CD$ , and from centre  $A$ , with the same opening of the compasses, draw a small arc  $G$ . Then from centre  $F$ , with a distance equal to  $DE$ , touch the arc in the point  $G$ . Join  $AG$ .  $BAG$  shall be the angle required.]



PROP. XXIV. THEOR.

*If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them of the other; the base of that which has the greater angle shall be greater than the base of the other.*

Let  $ABC$ ,  $DEF$  be two triangles,  
which have the two sides,  
 $AB$ ,  $AC$  equal to the two  $DE$ ,  $DF$ ,  
each to each, viz.  $AB$  equal to  $DE$ , and  $AC$  to  $DF$ ;  
but the angle  $BAC$  greater than the angle  $EDF$ ;  
the base  $BC$  is also greater than the base  $EF$ .

Of the two sides  $DE$ ,  $DF$ ,  
let  $DE$  be the side which is not greater than the other,  
and at the point  $D$ , in the straight line  $DE$ ,  
\* xxiii. 1. make \* the angle  $EDG$  equal to the angle  $BAC$ ;  
\* iii. 1. and make  $DG$  equal <sup>b</sup> to  $AC$  or  $DF$ ,  
and join  $EG$ ,  $GF$ .

Because  $AB$  is equal to  $DE$ ,  
and  $AC$  to  $DG$ ,  
the two sides  $BA$ ,  $AC$  are equal to the two  $ED$ ,  $DG$ ,  
each to each, and the angle  $BAC$  is equal to the angle  $EDG$ ;  
\* iv. 1. therefore the base  $BC$  is <sup>c</sup> equal to the base  $EG$ ;

and because  $DG$   
is equal to  $DF$ ,  
the angle  $DFG$  is  
equal<sup>d</sup> to the angle  
 $DGF$ ;  
but the angle  $DGF$  is  
greater than the angle  
 $EGF$ ;

therefore the angle

$DFG$  is greater than  $EGF$ ;  
and much more is the angle  $EFG$   
greater than the angle  $EGF$ ;  
and because the angle  $EFG$  of the triangle  $EFG$   
is greater than its angle  $EGF$ ,

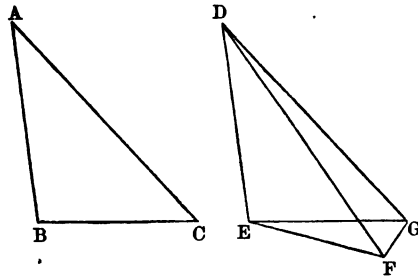
and that the greater<sup>e</sup> side is opposite to the greater angle; <sup>e</sup> xix. 1.

the side  $EG$  is therefore greater than the side  $EF$ ;

but  $EG$  is equal to  $BC$ ;

and therefore also  $BC$  is greater than  $EF$ .

Therefore, if two triangles, &c. Q. E. D.



<sup>d</sup> v. 1.

PROP. XXV. THEOR.

*If two triangles have two sides of the one equal to two sides of the other, each to each, but the base of the one greater than the base of the other; the angle also contained by the sides of that which has the greater base, shall be greater than the angle contained by the sides equal to them of the other.*

Let  $ABC$ ,  $DEF$  be two triangles,  
which have the two sides,  
 $AB$ ,  $AC$  equal to the two sides  $DE$ ,  $DF$ ,  
each to each, viz.  $AB$  equal to  $DE$ , and  $AC$  to  $DF$ ;  
but the base  $CB$  greater than the base  $EF$ ;  
the angle  $BAC$  is likewise greater than the angle  $EDF$ .

For, if it be not greater,  
it must either be equal to it, or less;

BOOK I. but the angle BAC is not equal to the angle EDF,

because then the base  
 \* iv. 1. BC would be equal<sup>a</sup>

to EF;

but it is not;

therefore the angle BAC

is not equal to the

angle EDF;

neither is it less;

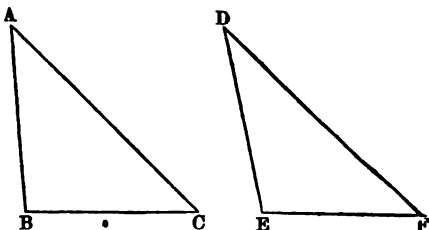
<sup>b</sup> xxiv. 1. because then the base BC would be less<sup>b</sup> than the base EF;  
 but it is not;

therefore the angle BAC is not less than the angle EDF;

and it was shown that it is not equal to it;

therefore the angle BAC is greater than the angle EDF.

Wherefore, if two triangles, &c. Q. E. D.



[It will be worth the student's while to cut out two triangles in cardboard, so as to exemplify the two last propositions.]

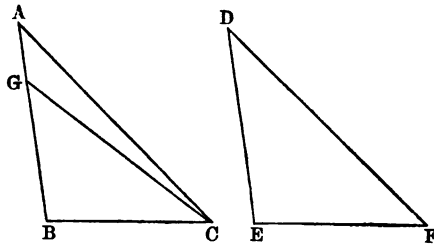
#### PROP. XXVI. THEOR.

*If two triangles have two angles of one equal to two angles of the other, each to each; and one side equal to one side, viz. either the sides adjacent to the equal angles, or the sides opposite to equal angles in each; then shall the other sides be equal, each to each; and also the third angle of the one to the third angle of the other.*

Let ABC, DEF be two triangles,  
 which have the angles ABC, BCA  
 equal to the angles DEF, EFD,  
 viz. ABC to DEF, and BCA to EFD;  
 also one side equal to one side;

CASE I. And first let those sides be equal which are adjacent to the angles that are equal in the two triangles;  
 viz. BC to EF;

the other sides shall  
be equal, each to  
each,  
viz. AB to DE,  
and AC to DF,  
and the third angle  
BAC to the third  
angle EDF.




---

For, if AB be not equal to DE,  
one of them must be the greater.

Let AB be the greater of the two,  
and make BG equal to DE,  
and join GC ;

---

Therefore, because BG is equal to DE,  
and BC to EF,  
the two sides GB, BC are equal to the two DE, EF,  
each to each ; and the angle GBC is equal to DEF ;  
therefore the base GC is equal <sup>a</sup> to the base DF,  
and the triangle GBC to the triangle DEF,  
and the other angles to the other angles,  
each to each, to which the equal sides are opposite ;  
therefore the angle GCB is equal to the angle DFE ;  
but DFE is, by the hypothesis, equal to the angle BCA,  
wherefore also the angle BCG is equal to the angle BCA,  
the less to the greater,  
which is impossible ;  
therefore AB is not unequal to DE,  
that is, it is equal to it ;  
and BC is equal to EF ;  
therefore the two AB, BC are equal to the two, DE, EF,  
each to each ; and the angle ABC is equal to DEF ;  
the base therefore AC is equal <sup>a</sup> to the base DF,  
and the third angle BAC to the third angle EDF.

---

<sup>a</sup> iv. 1.

**BOOK I. CASE II.** Next, let the sides which are opposite to equal angles,

in each triangle, be equal  
to one another,

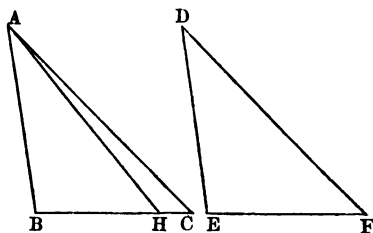
viz.  $AB$  to  $DE$ ;

likewise in this case, the  
other sides shall be  
equal,

$AC$  to  $DF$ ,

and  $BC$  to  $EF$ ;

and also the third angle  $BAC$  to the third  $EDF$ .



For, if  $BC$  be not equal to  $EF$ ,  
let  $BC$  be the greater of them,  
and make  $BH$  equal to  $EF$ ,  
and join  $AH$ ;

And because  $BH$  is equal to  $EF$ , and  $AB$  to  $DE$ ;  
the two  $AB$ ,  $BH$  are equal to the two  $DE$ ,  $EF$ , each to each;  
and they contain equal angles;  
therefore the base  $AH$  is equal to the base  $DF$ ,  
and the triangle  $ABH$  to the triangle  $DEF$ ,  
and the other angles shall be equal,  
each to each, to which the equal sides are opposite;  
therefore the angle  $BHA$  is equal to the angle  $EFD$ ;  
but  $EFD$  is equal to the angle  $BCA$ ;  
therefore also the angle  $BHA$  is equal to the angle  $BCA$ ;  
that is, the exterior angle  $BHA$  of the triangle  $AHC$ ,  
is equal to its interior and opposite angle  $BCA$ ;

<sup>b</sup> xvi. 1.

which is impossible <sup>b</sup>;

wherefore  $BC$  is not unequal to  $EF$ ,

that is, it is equal to it;

and  $AB$  is equal to  $DE$ ;

therefore the two,  $AB$ ,  $BC$  are equal to the two  $DE$ ,  $EF$ ,  
each to each; and they contain equal angles,

wherefore the base  $AC$  is equal to the base  $DF$ ,

and the third angle  $BAC$  to the third angle  $EDF$ .

Therefore, if two triangles, &c. Q. E. D.

PROP. XXVII. THEOR.

*If a straight line falling upon two other straight lines makes the alternate angles equal to one another, these two straight lines shall be parallel.*

Let the straight line EF,  
which falls upon the two straight lines AB, CD,  
make the alternate angles AEF, EFD equal to one another;  
AB is parallel to CD.

For, if it be not parallel,  
AB and CD being produced shall meet either towards B, D,  
or towards A, C:

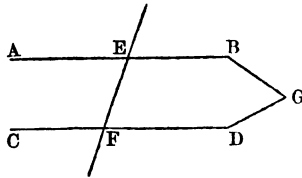
let them be produced and meet towards B, D, in the point G;  
therefore GEF is a triangle,

and its exterior angle AEF is greater<sup>a</sup> than the interior and  
opposite angle EFG;

but it is also equal to it,

which is impossible;

therefore AB and CD being  
produced do not meet to-  
wards B, D.



<sup>a</sup> xvi. 1

In like manner it may be demonstrated,  
that they do not meet towards A, C;  
but those straight lines which meet neither way,  
though produced ever so far,  
are parallel<sup>b</sup> to one another.

AB therefore is parallel to CD.

<sup>b</sup> 35 Def.

Wherefore, if a straight line, &c. Q. E. D.

BOOK I

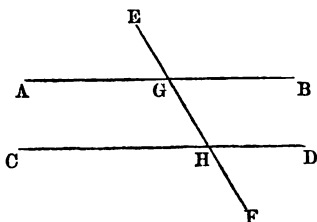
## PROP. XXVIII. THEOR.

*If a straight line falling upon two other straight lines makes the exterior angle equal to the interior and opposite upon the same side of the line ;*

*Or makes the interior angles upon the same side together equal to two right angles ;*

*The two straight lines shall be parallel to one another.*

Let the straight line EF,  
which falls upon the two straight lines AB, CD,  
make the exterior angle EGB,  
equal to the interior and opposite angle GHD upon the same  
side ;  
or make the interior angles on  
the same side,  
BGH, GHD, together equal to  
two right angles ;  
AB is parallel to CD.



- (1.) Because the angle EGB is equal to the angle GHD,  
<sup>a</sup> xv. 1. and the angle EGB equal <sup>a</sup> to the angle AGH,  
 the angle AGH is equal to the angle GHD ;  
 and they are the alternate angles ;  
<sup>b</sup> xxvii. 1. therefore AB is parallel <sup>b</sup> to CD.

- (2.) Again, because the angles BGH, GHD  
<sup>c</sup> By Hyp. are equal <sup>c</sup> to two right angles ;  
<sup>d</sup> xiii. 1. and that AGH, BGH are also equal <sup>d</sup> to two right angles ;  
 the angles AGH, BGH are equal to the angles BGH, GHD.  
 Take away the common angle BGH ;  
 therefore the remaining angle AGH is equal to  
 the remaining angle GHD ;  
 and they are alternate angles ;  
 therefore AB is parallel to CD.  
 Wherefore if a straight line, &c. Q. E. D.

PROP. XXIX. THEOR.

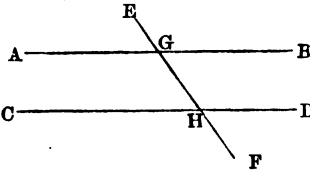
*If a straight line fall upon two parallel straight lines, it makes the alternate angles equal to one another.*

*And the exterior angle equal to the interior and opposite upon the same side.*

*And likewise the two interior angles upon the same side together equal to two right angles.*

Let the straight line EF fall upon the parallel straight lines AB, CD;

- (1.) the alternate angles AGH, GHD are equal to one another;
- (2.) and the exterior angle EGB is equal to the interior and opposite, upon the same side GHD;
- (3.) and the two interior angles BGH, GHD, upon the same side, are together equal to two right angles.



(1.) For, if AGH be not equal to GHD;  
one of them must be greater than the other;  
let AGH be the greater;  
and because the angle AGH is greater than the angle GHD;  
add to each of them the angle BGH;  
therefore the angles AGH, BGH are greater than the  
angles BGH, GHD;  
but the angles AGH, BGH are equal\* to two right angles; \* xiii. 1  
therefore the angles BGH, GHD are less than two right angles;  
but those straight lines which, with another straight line  
falling upon them, make the interior angles on the same  
side less than two right angles, do meet together if  
continually produced;  
therefore the straight lines AB, CD, if produced far enough,  
shall meet;  
but they never meet;  
since they are parallel by the hypothesis;  
therefore the angle AGH is not unequal to the angle GHD;

BOOK I. that is, it is equal to it;

xv. 1. (2.) but the angle AGH is equal<sup>b</sup> to the angle EGB;  
therefore likewise EGB is equal to GHD:

(3.) add to each of these the angle BGH;  
therefore the angles EGB, BGH are equal  
to the angles BGH, GHD;

\* xiii. 1. but EGB, BGH are equal<sup>c</sup> to two right angles;  
therefore also BGH, GHD are equal to two right angles.  
Wherefore, if a straight, &c. Q. E. D.

PROP. XXX. THEOR.

*Straight lines which are parallel to the same straight line are parallel to each other.*

Let AB, CD be each of them parallel to EF;  
AB is also parallel to CD.

Let the straight line GHK cut AB, EF, CD;

And because GHK cuts  
the parallel straight  
lines AB, EF;

\* xxix. 1. the angle AGH is equal<sup>a</sup>  
to the angle GHF.

Again, because the  
straight line GK

cuts the parallel straight lines EF, CD,  
the angle GHF is equal<sup>a</sup> to the angle GKD;

and it was shown that the angle AGK

is equal to the angle GHF;

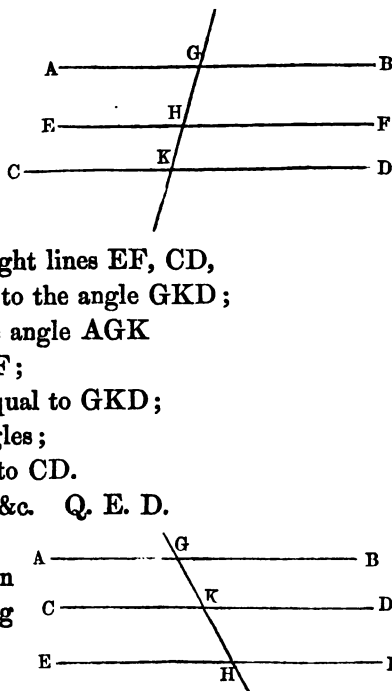
therefore, also, AGK is equal to GKD;

and they are alternate angles;

<sup>b</sup> xxvii. 1. therefore AB is parallel<sup>b</sup> to CD.

Wherefore straight lines, &c. Q. E. D.

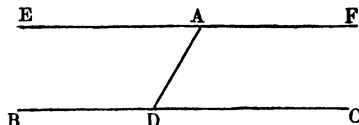
[Prove this Proposition  
also from the accompanying  
figure.]



## PROP. XXXI. PROB.

*To draw a straight line through a given point parallel to a given straight line.*

Let A be the given point;  
and BC the given straight line;  
it is required to draw a straight line through the point A,  
parallel to the straight  
line BC.



In BC take any point D,  
and join AD;  
and at the point A, in the straight line AD,  
make<sup>a</sup> the angle DAE equal to the angle ADC;  
and produce the straight line EA to F.

<sup>a</sup> xxiii. 1.

Because the straight line AD,  
which meets the two straight lines BC, EF,  
makes the alternate angles EAD, ADC equal to one another,  
EF is parallel<sup>b</sup> to BC.

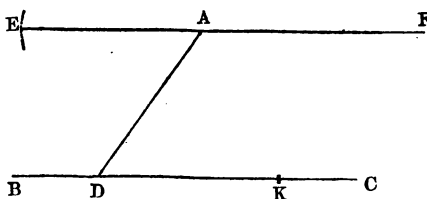
<sup>b</sup> xxvii. 1.

Therefore the straight line EAF is drawn  
through the given point A  
parallel to the given straight line BC.  
Which was to be done.

[In making the angle DAE equal to the angle ADC, use  
the method pointed out in the Note to Proposition XXIII.  
Thus:

Place one point of  
the compasses on A,  
and the other on D.

Then place the first  
point on K. Then  
with centre A, and



distance AD, draw an arc at E. Then from centre D, with  
distance AK, touch the arc in E. Join EA, and produce  
it to F.]

## BOOK I.

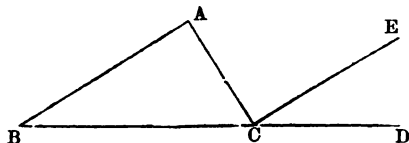
## PROP. XXXII. THEOR.

*If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles.*

*And the three interior angles of every triangle are equal to two right angles.*

Let  $ABC$  be a triangle,  
and let one of its sides  $BC$  be produced to  $D$ ;  
(1.) the exterior angle  $ACD$  is equal to  
the two interior and opposite angles  $CAB, ABC$ ;  
(2.) and the three interior angles of the triangle, viz.  $ABC, BCA, CAB$ ,  
are together equal to two right angles.

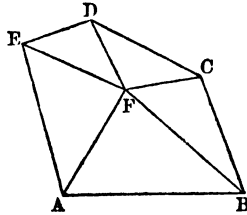
• xxxi. 1. Through the point  $C$   
draw  $CE$  parallel<sup>a</sup>  
to the straight line  
 $AB$ ,



- (1.) And because  $AB$  is parallel to  $CE$ ,  
and  $AC$  meets them,  
• xxix. 1. the alternate angles  $BAC, ACE$  are equal.<sup>b</sup>  
Again, because  $AB$  is parallel to  $CE$ ,  
and  $BD$  falls upon them,  
the exterior angle  $ECD$  is equal to  
the interior and opposite angle  $ABC$ ;  
but the angle  $ACE$  was shown to be equal to  $BAC$ ;  
therefore the whole exterior angle  $ACD$  is equal to  
the two interior and opposite angles  $CAB, ABC$ ;  
(2.) To these equals add the angle  $ACB$ ,  
and the angles  $ACD, ACB$  are equal to  
the three angles  $CBA, BAC, ACB$ ;  
• xviii. 1. but the angles  $ACD, ACB$  are equal<sup>c</sup> to two right angles;  
therefore also the angles  $CBA, BAC, ACB$   
are equal to two right angles.  
Wherefore if a side of a triangle, &c. Q. E. D.

COR. 1. All the interior angles of any rectilinear figure, BOOK I  
together with four right angles, are equal to  
twice as many right angles as the figure has sides.

For any rectilinear figure ABCDE  
can be divided into as many triangles as the figure has sides,  
by drawing straight lines from a  
point F within the figure to  
each of its angles.



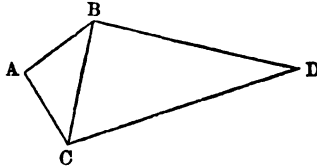
And, by the preceding Proposition,  
all the angles of these triangles are  
equal to twice as many right  
angles as there are triangles,  
that is, as there are sides of the figure;  
and the same angles are equal to the angles of the figure,  
together with the angles at the point F,  
which is the common vertex of the triangles;  
that is\*, together with four right angles.

Therefore all the angles of the figure,  
together with four right angles, are equal to  
twice as many right angles as the figure has sides.

\* 2 Cor.  
xv. 1.

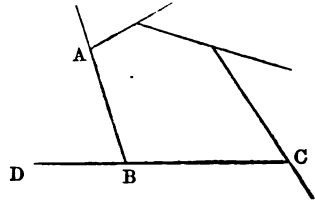
[The triangle furnishes an instance of this, for all its  
angles are together equal to two right angles; or all its  
angles, together with four right angles, are equal to six  
right angles, or twice as many right angles as the figure has  
sides. A square (see Def. 30.) and an oblong (see Def. 31.)  
are other obvious instances of this, for they have four right  
angles, and thus all their angles, together with four right  
angles, are together equal to eight right angles, that is, twice  
as many right angles as the figure has sides. The same  
may be exhibited in any four-sided figure by joining two of  
its opposite angular points. Thus:

The interior angles of ABDC  
are together equal to four right  
angles, for they are made up  
of all the angles of the two  
triangles into which the figure  
is divided, that is, twice two right angles.]



BOOK I. COR. 2. All the exterior angles of any rectilinear figure are together equal to four right angles.

Because every interior angle  $ABC$ ,  
 with its adjacent exterior angle  $ABD$ ,  
 is equal<sup>b</sup> to two right angles;  
 therefore all the interior together  
 with all the exterior angles  
 of the figure,  
 are equal to twice as many right  
 angles as there are sides of  
 the figure;  
 that is, by the foregoing corollary,  
 they are equal to all the interior angles of the figure,  
 together with four right angles;  
 therefore all the exterior angles are equal to four right angles.



[This also may be advantageously illustrated in the triangle and the square.]

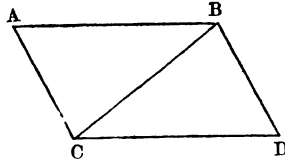
PROP. XXXIII. THEOR.

*The straight lines which join the extremities of two equal and parallel straight lines, towards the same parts, are also themselves equal (1.) and (2.) parallel.*

Let AB, CD be equal and parallel straight lines,  
and joined towards the same parts by the straight lines AC,  
BD,  
AC, BD are also equal and parallel.

Join BC ;

(1.) And because AB is parallel  
to CD,  
and BC meets them,  
the alternate angles ABC, BCD  
are equal<sup>a</sup>;



<sup>a</sup> xxix. 1.

and because AB is equal to CD,  
and BC common to the two triangles, ABC, DCB,  
the two sides AB, BC are equal to the two DC, CB ;  
and the angle ABC is equal to the angle BCD ;  
therefore the base AC is equal<sup>b</sup> to the base BD,

(2.) And the triangle ABC to the triangle BCD,  
and the other angles to the other angles<sup>b</sup>,  
each to each, to which the equal sides are opposite ;  
therefore the angle ACB is equal to the angle CBD ;  
and because the straight line BC meets the two straight  
lines AC, BD,  
and makes the alternate angles ACB, CBD equal to one  
another,

<sup>b</sup> iv. 1.

AC is parallel<sup>c</sup> to BD ;

<sup>c</sup> xxvii. 1.

and it was shown to be equal to it.

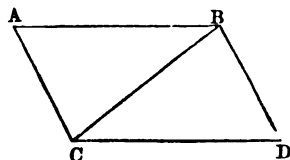
Therefore, straight lines, &c. Q. E. D.

## BOOK I

## PROP. XXXIV. THEOR.

*The opposite (1.) sides and (2.) angles of parallelograms are equal to one another, and (3.) the diameter bisects them, that is, divides them into two equal parts.*

Let ACDB be a parallelogram,  
of which BC is a diameter ;  
the opposite (1.) sides and (2.) angles of the figure  
are equal to one another ;  
and (3.) the diameter BC bisects it.



(1.) Because AB is parallel to  
CD,

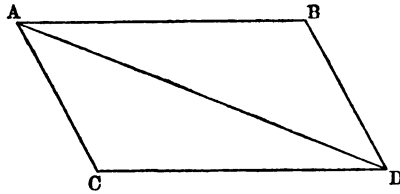
and BC meets them,

- <sup>a</sup> xxix. 1. the alternate angles ABC, BCD are equal to one another<sup>a</sup> ;  
and because AC is parallel to BD, and BC meets them,  
the alternate angles ACB, CBD are equal<sup>a</sup> to one another ;  
wherefore the two triangles ABC, CBD  
have two angles ABC, BCA in one,  
equal to two angles BCD, CBD in the other,  
each to each, and one side BC common to the two triangles,  
which is adjacent to their equal angles ;  
therefore their other sides shall be equal,  
each to each,  
and the third angle of the one  
<sup>b</sup> xxvi. 1. to the third angle of the other<sup>b</sup> ,  
viz. the side AB to the side CD,  
and AC to BD,

(2.) And the angle BAC equal to the angle BDC :  
And because the angle ABC is equal to the angle BCD,  
and the angle CBD to the angle ACB,  
the whole angle ABD is equal to the whole angle ACD :  
And the angle BAC has been shown to be equal to the  
angle BDC ;  
therefore the opposite sides and angles of parallelograms are  
equal to one another ;

(3.) Also, the diameter bisects them;  
for AB being equal to CD, and BC common,  
the two AB, BC are equal to  
the two DC, CB, each to each;  
and the angle ABC is equal to the angle BCD;  
therefore the triangle ABC is equal<sup>a</sup> to the triangle BCD, <sup>a</sup> iv. 1.  
and the diameter BC divides the parallelogram ACDB into  
two equal parts. Q. E. D.

[The same should, for  
the sake of practice, be  
proved with the accom-  
panying figure.]



PROP. XXXV. THEOR.

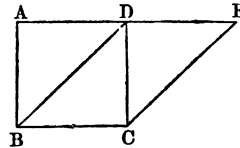
*Parallelograms upon the same base, and between the same  
parallels, are equal to one another.*

Let the parallelograms ABCD, EBCF  
be upon the same base BC,  
and between the same parallels AF, BC;  
the parallelogram ABCD shall be equal to  
the parallelogram EBCF.

See the  
2nd and  
3rd figures.

CASE I. If the sides AD, DF of  
the parallelograms ABCD,  
DBCF,

opposite to the base BC, be termi-  
nated in the same point D;



it is plain that each of the parallelograms is double<sup>a</sup> of the <sup>a</sup> xxxiv. 1.  
triangle BDC;

and they are therefore equal to one another.<sup>b</sup>

<sup>b</sup> 6 Ax.

CASE II. But, if the sides AD, EF, opposite to the base  
BC of the parallelograms ABCD, EBCF,  
be not terminated in the same point;

BOOK I. then, because ABCD is a parallelogram,

• 6 Ax. AD is equal<sup>a</sup> to BC :

for the same reason EF is equal to BC ;

• 1 Ax. wherefore AD is equal<sup>b</sup> to EF ;

and DE is common ;

therefore the whole, or the remainder,

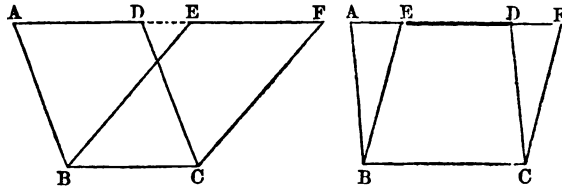
• 2 or 3 AE is equal<sup>c</sup> to the whole, or the remainder DF ;

Ax.

AB also is equal to DC ;

and the two EA, AB are therefore equal to the two FD, DC, each to each ;

• xxix. 1. and the exterior angle FDC is equal<sup>d</sup> to the interior EAB ;  
therefore the base EB is equal to the base FC,



• iv. 1. and the triangle EAB equal<sup>e</sup> to the triangle FDC.

Take the triangle FDC from the trapezium ABCF ;

and from the same trapezium take the triangle EAB :

• 3 Ax. the remainders therefore are equal<sup>f</sup>,

that is,

the parallelogram ABCD is equal to the parallelogram EBCF.

Therefore parallelograms upon the same base, &c. Q. E. D.

## PROP. XXXVI. THEOR.

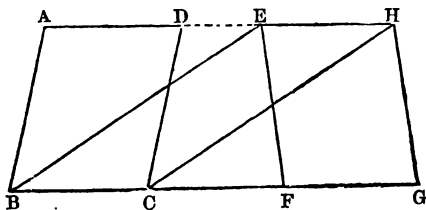
## BOOK I.

*Parallelograms upon equal bases, and between the same parallels, are equal to one another.*

Let ABCD, EFGH be parallelograms  
upon equal bases  
BC, FG,  
and between the same parallels

AH, BG;  
the parallelogram  
ABCD is equal  
to EFGH.

Join BE, CH;



And because BC is equal to FG,

and FG to <sup>a</sup> EH,

<sup>a</sup> xxxiv. 1.

BC is equal to EH;

and they are parallels,

and joined towards the same parts

by the straight lines BE, CH.

But straight lines which join equal and parallel straight lines

towards the same parts, are themselves equal and parallel<sup>b</sup>; <sup>b</sup> xxxiii. 1.

therefore EB, CH are both equal and parallel,

and EBCH is a parallelogram;

and it is equal<sup>c</sup> to ABCD,

<sup>c</sup> xxxv. 1.

because it is upon the same base BC,

and between the same parallels BC, AD.

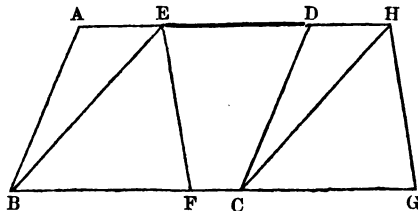
For the like reason,

the parallelogram EFGH is equal to the same EBCH.

Therefore also the parallelogram ABCD is equal to EFGH.

Wherefore parallelograms, &c. Q. E. D.

[The figure may also  
be drawn like the annexed,  
and the Proposition should be proved  
from it.]

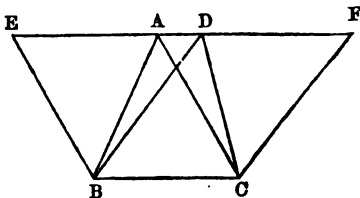


## BOOK I.

## PROP. XXXVII. THEOR.

*Triangles upon the same base, and between the same parallels, are equal to one another.*

Let the triangles ABC, DBC be upon the same base BC, and between the same parallels AD, BC :  
The triangle ABC is equal to the triangle DBC.



Produce AD both ways to the points E, F,

- \* xxxi. 1 and through B draw \* BE parallel to CA ;  
and through C draw CF parallel to BD.

Therefore each of the figures EBCA, DBCF is a parallelogram ;

- <sup>b</sup> xxxv. 1. and EBCA is equal <sup>b</sup> to DBCF,  
because they are upon the same base BC,  
and between the same parallels BC, EF ;  
and the triangle ABC is the half of the parallelogram, EBCA,  
<sup>c</sup> xxxiv. 1. because the diameter AB bisects <sup>c</sup> it ;  
and the triangle DBC is the half of the parallelogram DBCF,  
because the diameter DC bisects it :  
<sup>d</sup> 7 Ax. but the halves of equal things are equal <sup>d</sup> ;  
therefore the triangle ABC is equal to the triangle DBC.  
Wherefore triangles, &c. Q. E. D.

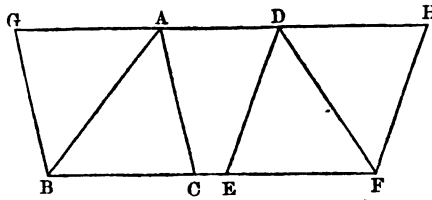
PROP. XXXVIII. THEOR.

*Triangles upon equal bases, and between the same parallels, are equal to one another.*

Let the triangles ABC, DEF be upon equal bases BC, EF, and between the same parallels BF, AD:  
The triangle ABC is equal to the triangle DEF.

Produce AD both ways to the points G, H, and through B draw BG parallel <sup>a</sup> to CA, and through F draw FH parallel to ED:

<sup>a</sup> xxxi. 1.



Then each of the figures GBCA, DEFH is a parallelogram;

and they are equal to <sup>b</sup> one another, because they are upon equal bases BC, EF, and between the same parallels BF, GH; and the triangle ABC is the half <sup>c</sup> of the parallelogram GBCA, because the diameter AB bisects it; and the triangle DEF is the half <sup>c</sup> of the parallelogram DEFH, because the diameter DF bisects it:  
But the halves of equal things are equal <sup>d</sup>;  
therefore the triangle ABC is equal to the triangle DEF.  
Wherefore triangles, &c. Q. E. D.

<sup>b</sup> xxxvi. 1.

<sup>c</sup> xxxiv. 1.

<sup>d</sup> 7 Ax.

## BOOK I.

## PROP. XXXIX. THEOR.

*Equal triangles upon the same base, and upon the same side of it, are between the same parallels.*

Let the equal triangles  $ABC$ ,  $DBC$  be upon the same base  $BC$ ,  
and upon the same side of it;  
they are between the same parallels.

Join  $AD$ ;  
 $AD$  is parallel to  $BC$ ;

For, if it is not,

<sup>a</sup> xxxi. 1. through the point  $A$  draw <sup>a</sup>  $AE$  parallel to  $BC$ ,  
and join  $EC$ :

<sup>b</sup> xxxvii. 1. The triangle  $ABC$  is equal <sup>b</sup> to  
the triangle  $EBC$ ,  
because it is upon the same base  $BC$ ,  
and between the same parallels  
 $BC$ ,  $AE$ :

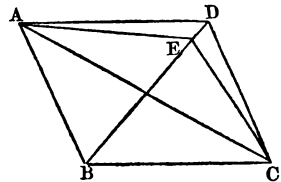
But the triangle  $ABC$  is equal to  
the triangle  $BDC$ ;

therefore also the triangle  $BDC$  is equal to the triangle  $EBC$ ,  
the greater to the less,  
which is impossible.

Therefore  $AE$  is not parallel to  $BC$ .

In the same manner it can be demonstrated,  
that no other line but  $AD$  is parallel to  $BC$ ;  
 $AD$  is therefore parallel to it.

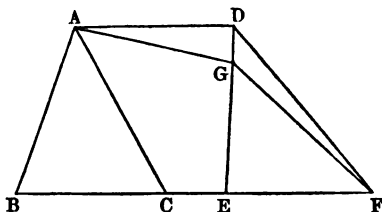
Wherefore equal triangles upon, &c. Q. E. D.



## PROP. XL. THEOR.

*Equal triangles upon equal bases, in the same straight line, and towards the same parts, are between the same parallels.*

Let the equal triangles ABC, DEF  
be upon equal bases BC, EF, in the same straight line BF,  
and towards the same  
parts;  
they are between the  
same parallels.



Join AD:  
AD is parallel to BC:

For, if it is not,  
through A draw <sup>a</sup> AG parallel to BF,  
and join GF:

<sup>a</sup> xxxi. 1.

The triangle ABC is equal <sup>b</sup> to the triangle GEF,  
because they are upon equal bases BC, EF,  
and between the same parallels BF, AG.

<sup>b</sup> xxxviii. 1.

But the triangle ABC is equal to the triangle DEF;  
therefore also the triangle DEF is equal to the triangle GEF,  
the greater to the less,  
which is impossible:

Therefore AG is not parallel to BF:  
And in the same manner it can be demonstrated  
that there is no other parallel to it but AD,  
AD is therefore parallel to BF.

Wherefore equal triangles, &c. Q. E. D.

BOOK I.

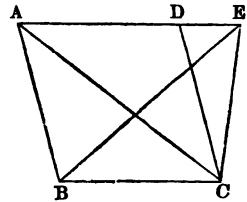
## PROP. XLI. THEOR.

*If a parallelogram and triangle be upon the same base, and between the same parallels :*

*The parallelogram shall be double of the triangle.*

Let the parallelogram ABCD and the triangle EBC  
be upon the same base BC,  
and between the same parallels BC, AE ;  
the parallelogram ABCD is double of  
the triangle EBC.

Join AC ;



\* xxxvii. 1. then the triangle ABC is equal\* to  
the triangle EBC,

because they are upon the same base BC,  
and between the same parallels BC, AE.

<sup>b</sup> xxxiv. 1. But the parallelogram ABCD is double<sup>b</sup> of the triangle ABC,  
because the diameter AC divides it into two equal parts ;  
wherefore ABCD is also double of the triangle EBC.  
Therefore, if a parallelogram, &c. Q. E. D.

PROP. XLII. PROB.

*To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.*

Let  $ABC$  be the given triangle,  
and  $D$  the given rectilineal angle.  
It is required to describe a parallelogram  
that shall be equal to the given triangle  $ABC$ ,  
and have one of its angles equal to  $D$ .

Bisect <sup>a</sup>  $BC$  in  $E$ ,  
join  $AE$ ,  
and at the point  $E$  in the straight line  $EC$ ,  
make <sup>b</sup> the angle  $CEF$  equal to  $D$ ;  
and through  $A$  draw <sup>c</sup>  $AG$  parallel to  $EC$ ,  
and through  $C$  draw  $CG$  <sup>c</sup> parallel to  $EF$ :

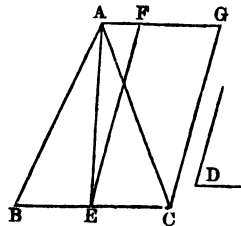
<sup>a</sup> x. 1.

<sup>b</sup> xxiii. 1.

<sup>c</sup> xxxi. 1.

Therefore  $FECG$  is a parallelogram;  
And because  $BE$  is equal to  $EC$ ,  
the triangle  $ABE$  is likewise equal <sup>d</sup> to  
the triangle  $AEC$ ,  
since they are upon equal bases  $BE$ ,  
 $EC$ ;

<sup>d</sup> xxxviii. 1.



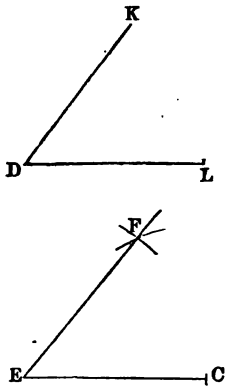
and between the same parallels  $BC$ ,  $AG$ :  
therefore the triangle  $ABC$  is double of the triangle  $AEC$ .  
And the parallelogram  $FECG$   
is likewise double <sup>e</sup> of the triangle  $AEC$ ,  
because it is upon the same base,  
and between the same parallels:  
Therefore the parallelogram  $FECG$   
is equal to the triangle  $ABC$ ,  
and it has one of its angles  $CEF$   
equal to the given angle  $D$ ;  
wherefore there has been described  
a parallelogram  $FECG$  equal to a given triangle  $ABC$ ,

<sup>e</sup> xli. 1.

**BOOK I** having one of its angles CEF equal to the given angle D.  
Which was to be done.

[In making the angle CEF equal to D, use the method pointed out in the Note to Prop. XXIII. Thus:

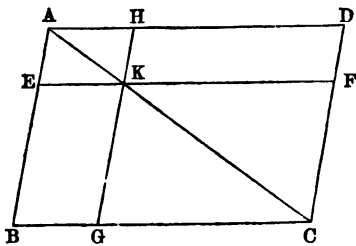
Mark off DK, DL equal to each other, and EC equal to either of these, and with centre E, and distance EC, describe the arc F. Then with centre C, and distance equal to KL, touch the arc F. Join FE. The angle FEC will be the required angle.]



#### PROP. XLIII. THEOR.

*The complements of the parallelograms, which are about the diameter of any parallelogram, are equal to one another.*

Let ABCD be a parallelogram,  
of which the diameter is AC,  
and EH, FG the parallelograms about AC,  
that is through which AC passes,  
and BK, KD, the other parallelograms  
which make up the whole  
figure ABCD,  
which are therefore called  
the complements:  
The complement BK is  
equal to the complement  
KD.



Because ABCD is a parallelogram,  
and AC its diameter,

- \* xxxiv. 1. the triangle ABC is equal \* to the triangle ADC:  
And, because EKHA is a parallelogram,

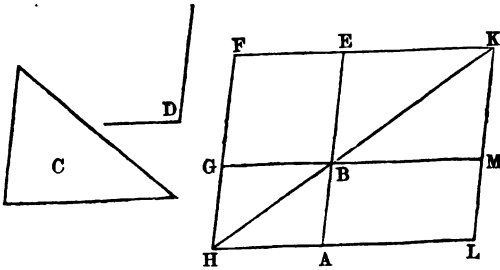
the diameter of which is AK,  
 the triangle AEK is equal to the triangle AHK :  
 By the same reason,  
 the triangle KGC is equal to the triangle KFC :  
 Then, because the triangle AEK is equal to AHK,  
 and the triangle KGC to KFC ;  
 the triangle AEK, together with the triangle KGC,  
 is equal to the triangle AHK,  
 together with the triangle KFC :  
 But the whole triangle ABC is equal to the whole ADC ;  
 therefore the remaining complement BK  
 is equal to the remaining complement KD.  
 Wherefore the complements, &c. Q. E. D.

PROP. XLIV. PROB.

*To a given straight line to apply a parallelogram, which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.*

Let AB be the given straight line,  
 and C the given triangle,  
 and D the given rectilineal angle.  
 It is required to apply to the straight line AB  
 a parallelogram equal to the triangle C,  
 and having an angle equal to D.

Make\* the parallelogram BEFG equal to the triangle C, • xlii. 1.  
 and having the angle EBG equal to the angle D,



**BOOK I.** so that BE be in the same straight line with AB,  
and produce FG to H;

- <sup>b</sup> xxxi. 1. and through A draw <sup>b</sup> AH parallel to BG or EF,  
and join HB.

---

Then because the straight line HF  
falls upon the parallels AH, EF,

- <sup>c</sup> xxix. 1. the angles AHF, HFE are equal <sup>c</sup> to two right angles;  
wherefore BHF, HFE are less than two right angles:

But straight lines which with another straight line  
make the interior angles upon the same side  
less than two right angles,

- <sup>d</sup> 12 Ax. do meet <sup>d</sup> if produced far enough:

Therefore HB, FE shall meet if produced;  
let them meet in K,

and through K draw KL, parallel to EA or FH,  
and produce HA, GB to the points L, M:

Then HLKF is a parallelogram,  
of which the diameter is HK;

and AG, ME are the parallelograms about HK;  
and LB, BF are the complements;

- <sup>e</sup> xliii. 1. therefore LB is equal <sup>e</sup> to BF:

But BF is equal to the triangle C;

wherefore LB is equal to the triangle C;

- <sup>f</sup> xv. 1. and because the angle GBE is equal <sup>f</sup> to the angle ABM,  
and likewise to the angle D;

the angle ABM is equal to the angle D:

Therefore the parallelogram LB is applied to the straight  
line AB,

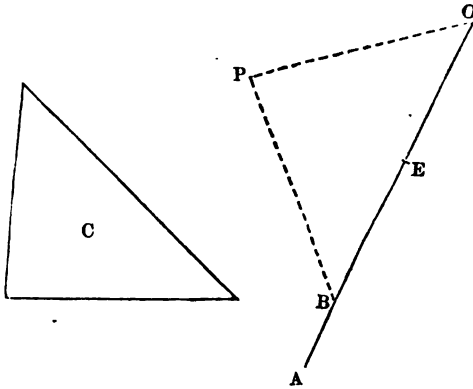
is equal to the triangle C,

and has the angle ABM equal to the angle D.

Which was to be done.

---

[In order to draw this figure correctly, a triangle must  
first be placed upon the line AB produced, equal to C.  
Thus:



Measure BO equal to one of the sides of C, and, as in the Note to Prop. XXIII., make the other sides OP and BP equal respectively to the two other sides of the triangle. Then proceed with the triangle PBO just as the figure in Prop. XLII. was drawn.]

## BOOK I.

## PROP. XLV. PROB.

*To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.*

Let ABCD be the given rectilineal figure,  
and E the given rectilineal angle.

It is required to describe a parallelogram equal to ABCD,  
and having an angle equal to E.

Join DB,

- xlii. 1. and describe <sup>a</sup> the parallelogram FH  
equal to the triangle ADB,  
and having the angle HKF equal to the angle E,  
and to the straight line GH

- xliv. 1. apply <sup>b</sup> the parallelogram GM equal to the triangle DBC,  
having the angle GHM equal to the angle E;

and because the angle E is equal to each of the angles FKH,  
GHM,

the angle FKH is equal to GHM:

add to each of these the angle KHG;

therefore the angles FKH, KHG are equal to the angles  
KHG, GHM;

but FKH, KHG

- xxix. 1. are equal <sup>c</sup> to two  
right angles;

Therefore also

KHG, GHM

are equal to two  
right angles;

and because at the point H in the straight line GH,

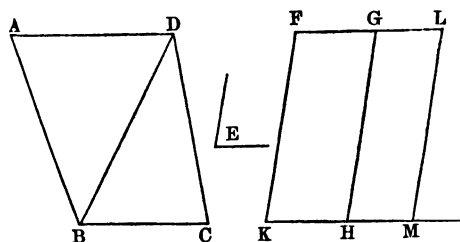
the two straight lines KH, HM,

upon the opposite sides of it,

make the adjacent angles equal to two right angles,

- <sup>d</sup> xiv. 1. KH is in the same straight line <sup>d</sup> with HM,

and because the straight line HG



meets the parallels KM, FG,

the alternate angles MHG, HGF are equal °:

Add to each of these the angle HGL :

Therefore the angles MHG, HGL

are equal to the angles HGF, HGL :

But the angles MHG, HGL are equal ° to two right angles ;

wherefore also HGF, HGL are equal to two right angles,

and FG is therefore in the same straight line with GL ;

and because KF is parallel to HG,

and HG to ML ;

KF is parallel ° to ML ;

• xxx. 1.

and KM, FL are parallels ;

wherefore KFLM is a parallelogram ;

and because the triangle ABD is equal to the parallelogram

HF,

and the triangle DBC to the parallelogram GM ;

the whole figure ABCD is equal to the whole KFLM ;

therefore the parallelogram KFLM has been described

equal to the given rectilineal figure ABCD,

having the angle FKM equal to the given angle E.

Which was to be done.

COR. From this it is manifest how

to a given straight line to apply a parallelogram,

which shall have an angle equal to a given rectilineal angle,

and shall be equal to a given rectilineal figure,

viz. by applying<sup>b</sup> to the given straight line a parallelogram

<sup>b</sup> xliv. 1.

equal to the first triangle ABD,

and having an angle equal to the given angle.

[The drawing the figure of this Proposition may advantageously be omitted at the first reading. Indeed the process of drawing it being a mere repetition of the processes employed in XLII. and XLIV. may be omitted altogether, if the figures in XLII. and XLIV. have been once carefully drawn.]

## BOOK I.

## PROP. XLVI. PROB.

*To describe a square upon a given straight line.*

Let AB be the given straight line ;  
it is required to describe a square upon AB.

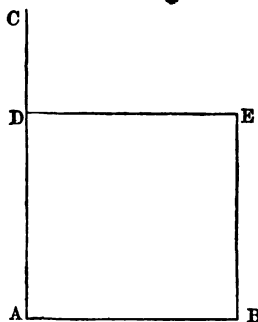
- 
- xi. 1. From the point A draw <sup>a</sup> AC at right angles to AB ;
  - iii. 1. and make <sup>b</sup> AD equal to AB,
  - xxxi. 1. and through the point D draw DE parallel<sup>c</sup> to AB,  
and through B draw BE parallel to AD ;
- 

Therefore ADEB is a parallelogram :

- xxxiv. 1. whence AB is equal<sup>d</sup> to DE,  
and AD to BE :

But BA is equal to AD ;  
therefore the four straight lines  
BA, AD, DE, EB  
are equal to one another,  
and the parallelogram ADEB is  
equilateral,  
likewise all its angles are right  
angles ;

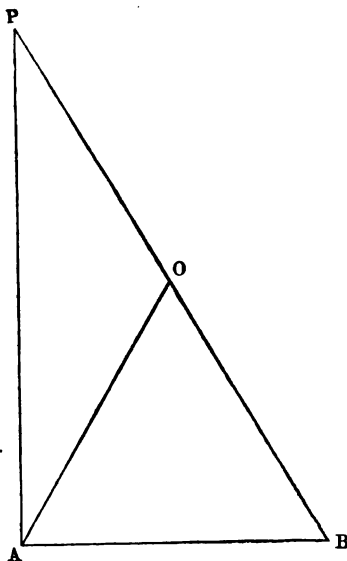
because the straight line AD meet-  
ing the parallels AB, DE,



- xxix. 1. the angles BAD, ADE are equal<sup>e</sup> to two right angles:  
but BAD is a right angle ;  
therefore also ADE is a right angle ;  
but the opposite angles of parallelograms are equal<sup>d</sup> ;  
therefore each of the opposite angles ABE, BED is a right  
angle ;  
wherefore the figure ADEB is rectangular,  
and it has been demonstrated that it is equilateral ;  
it is therefore a square,  
and it is described upon the given straight line AB.  
Which was to be done.

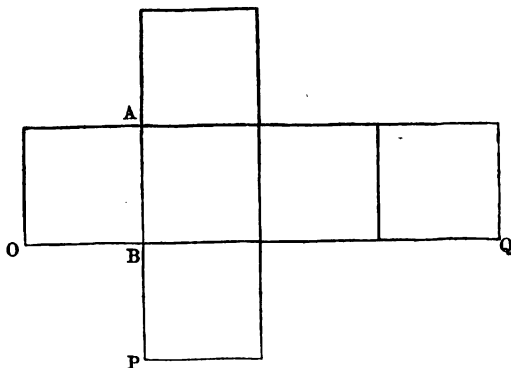
**COR.** Hence every parallelogram that has one right angle BOOK I. has all its angles right angles.

[As it is convenient to erect a perpendicular at the end of a line without producing it, use the following method: Take any point B on the line required. On AB construct an equilateral triangle AOB. Produce BO to P cutting off OP equal to OB. PA shall be perpendicular to AB.



The proof that it is so is as follows: the angle P is equal to the angle OAP, and the angle B is equal to the angle OAB. Add these equals, and the angles P and B together are equal to the angles PAO and OAB, or to PAB. But the angle PAB, together with the angles P and B, are equal to two right angles. Therefore the angle PAB is a right angle.

Practise constructing squares, as follows :



**BOOK I** Upon  $AB$  construct a square, and upon each of its sides construct squares, and upon one of the exterior lines of the figure construct another square.

Cut the figure out in cardboard, and let the squares be moveable on the sides of the first constructed square as hinges, and the last drawn square upon the side on which it was constructed: turn them till the points  $O, P, Q$  coincide. The solid figure formed is called a cube, and is the second of the five regular solids.]

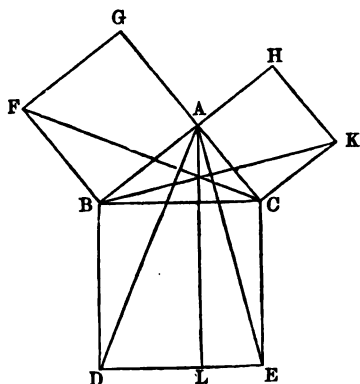
### PROP. XLVII. THEOR.

*In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the squares described upon the sides which contain the right angle.*

Let  $ABC$  be a right-angled triangle  
having the right angle  $BAC$ ;  
the square described upon the side  $BC$  is equal to  
the squares described upon  $BA, AC$ .

- \* xlv. 1. On  $BC$  describe<sup>a</sup> the square  $BDEC$ ,  
and on  $BA, AC$  the squares  $GB, HC$ ;  
b xxxi. 1. and through  $A$  draw<sup>b</sup>  $AL$  parallel to  $BD$ , or  $CE$ ,  
and join  $AD, FC$ ;

- Then, because each of the  
angles  $BAC, BAG$  is  
a right angle<sup>c</sup>  
• 30 Def. the two straight lines  $AC,$   
 $AG$ , upon the opposite  
sides of  $AB$ ,  
make with it at the point  $A$ ,  
the adjacent angles equal to  
two right angles;  
therefore  $CA$  is in the same  
straight line<sup>d</sup> with  $AG$  :



for the same reason, AB and AH are in the same straight line; BOOK I

and because the angle DBC is equal to the angle FBA,  
each of them being a right angle,

add to each the angle ABC,

and the whole angle DBA is equal to the whole FBC,

• 2 Ax.

and because the two sides AB, BD

are equal to the two FB, BC,

each to each,

and the angle DBA equal  
to the angle FBC;

therefore the base AD is  
equal to the base FC,

and the triangle ABD to  
the triangle FBC.

Now the parallelogram BL  
is double of the tri-  
angle ABD,

because they are upon the  
same base BD,

and between the same pa-  
rallels, BD, AI;

and the square GB is double  
of the triangle FBC,

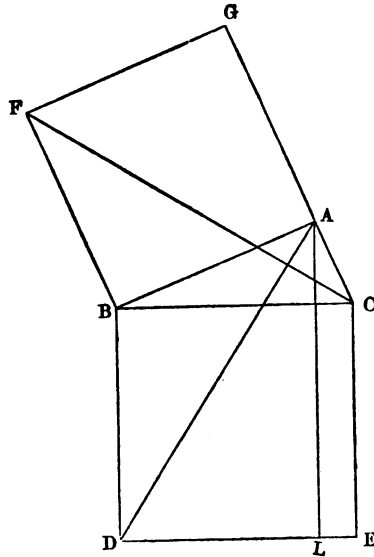
because these are also upon  
the same base FB,

and between the same  
parallels FB, GC.

But the doubles of equals  
are equal to one  
another:

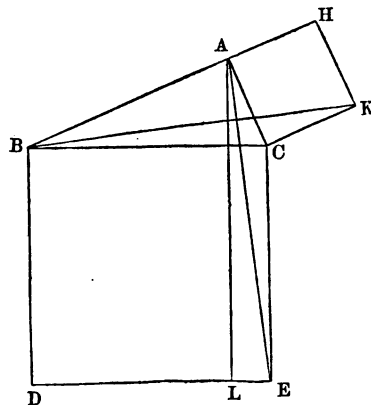
Therefore the parallelogram  
BL is equal to the  
square GB:

And, in the same manner,  
by joining AE, BK it is  
demonstrated,



iv. 1.

xli. 1.



6 Ax.

**BOOK I** that the parallelogram CL is equal to the square HC ;  
 Therefore the whole square BDEC is equal to the two  
 squares GB, HC ;  
 and the square BDEC is described upon the straight line BC,  
 and the squares GB, HC upon BA, AC :  
 Wherefore the square upon the side BC is equal to the  
 squares upon the sides BA, AC.  
 Therefore, in any right-angled triangle, &c. Q. E. D.

PROP. XLVIII. THEOR.

*If the square described upon one of the sides of a triangle, be equal to the squares described upon the other two sides of it ; the angle contained by these two sides is a right angle.*

If the square described upon BC,  
 one of the sides of the triangle ABC,  
 be equal to the squares upon the other side BA, AC,  
 the angle BAC is a right angle.

\* xi 1. From the point A draw <sup>a</sup> AD at right angles to AC,  
 and make AD equal to BA,  
 and join DC.

Then, because DA is equal to AB,  
 the square of DA is equal to the  
 square of AB.

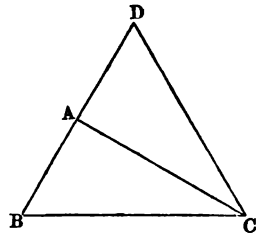
To each of these add  
 the square of AC ;

therefore the squares of DA, AC

are equal to the squares of BA, AC.

<sup>b</sup> xlvii. 1. But the square of DC is equal <sup>b</sup> to the squares of DA, AC,  
 because DAC is a right angle ;

By Hyp. and the square of BC is equal to the squares of BA, AC ;  
 therefore the square of DC is equal to the square of BC ;  
 and therefore also the side DC is equal to the side BC.  
 And because the side DA is equal to AB,



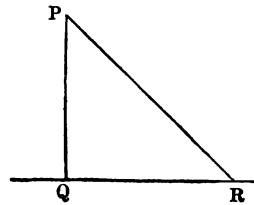
and AC common to the two triangles DAC, BAC,  
the two DA, AC are equal to the two BA, AC;  
and the base DC is equal to the base BC;  
therefore the angle DAC is equal<sup>o</sup> to the angle BAC;  
but DAC is a right angle;  
therefore also BAC is a right angle.  
Therefore, if the square, &c. Q. E. D. • iii. 1.

[The easiest mode of drawing this figure is by producing the line BA to D. It must be remembered, that though AD does lie in the same straight line with AB, this is in the present case only a consequence of its being drawn at right angles to AC.]

## DEDUCTIONS FROM BOOK I.

1. To prove that the perpendicular is the shortest straight line from a given point to a given straight line.

Take any other straight line  $PR$ , and prove by means of Propositions, I. 32. and I. 19.



2. Through a given point to draw a straight line, which shall cut off equal segments from two given intersecting straight lines.

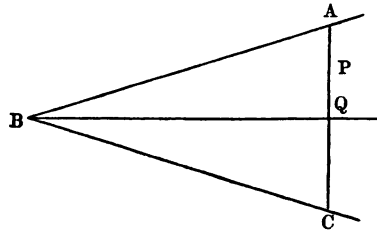
Bisect the angle  $ABC$

by the straight line  $BQ$ .

Draw  $PQ$  perpendicular to it, and produce it.

$BA$  is equal to  $BC$ .

Prove by means of I. 26.

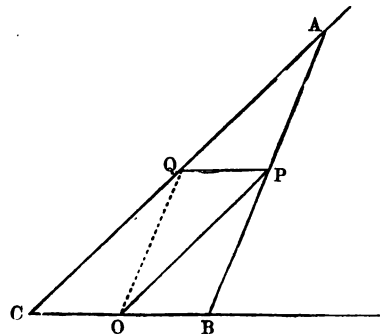


3. Through a given point to draw a straight line meeting two given straight lines which shall be bisected at the point.

Draw  $PO$ ,  $PQ$  parallel to the two given lines.

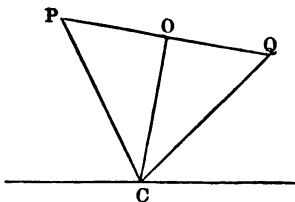
Join  $OQ$ , and draw  $APB$  parallel to it.

$AP$  and  $PB$  may be proved to be each equal to  $OQ$  by I. 34.



4. To find the point in a given straight line, from which two equal straight lines can be drawn to two given points.

Join  $PQ$ . Bisect  $PQ$  in  $O$ .  
 Draw  $OC$  at right angles to it.  
 $CP$  shall be equal to  $CQ$ .  
 Prove by I. 4.

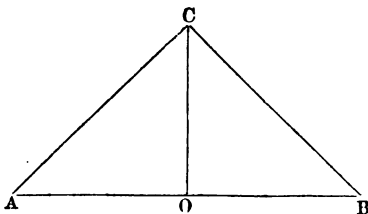


5. Upon a given straight line to construct an isosceles right angled triangle, whose right angle shall be subtended by the given line.

Bisect  $AB$  in  $O$ .  
 Draw  $OC$  at right angles to it, equal to  $OA$  or  $OB$ .

Join  $CA, CB$ .

Prove  $AC$  equal to  $CB$  by I. 4.



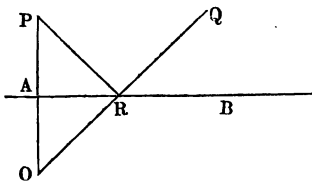
Then prove  $ACB$  to be a right angle by I. 32., showing that  $A$  and  $B$  are each half a right angle.

6. From two given points on the same side of a given straight line, to draw two straight lines which shall meet in it, and make equal angles with it.

From either point as  $P$  drop a perpendicular, and produce it to  $O$ , making  $AO$  equal to  $AP$ .

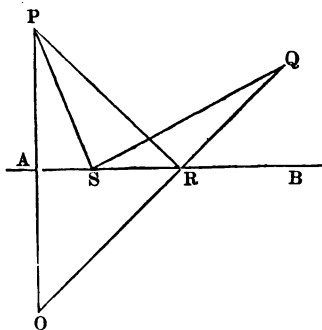
Join  $OQ$ . The lines  $PR, RQ$  shall be those required.

Prove by I. 4., the angle  $PRA$  equal to the angle  $ORA$ , and equal to  $QRB$  by means of I. 15.



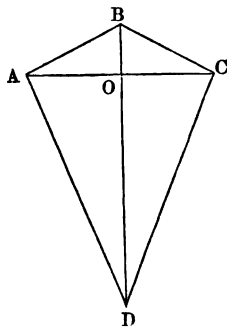
- BOOK I** 7. In the figure of the last Proposition prove that the sum of  $PR$ ,  $RQ$  is less than the sum of any other straight lines drawn from the points  $P$ ,  $Q$  to meet in the straight line  $AB$ .

This may be done after the addition of one line to the figure, by means of I. 20.



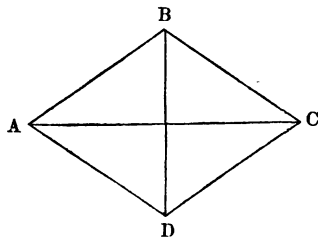
8. In a quadrilateral figure which has two pairs of adjacent sides equal, to prove that the diagonals are at right angles to each other.

Use I. 8. with reference to the triangles  $BAD$ ,  $BCD$ , and then proceed by means of I. 4. to prove  $AOB$  a right angle.



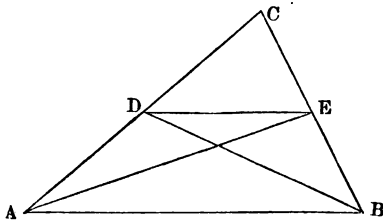
9. In a rhombus prove that the diagonals bisect each other at right angles.

Use the same Propositions as in the above.

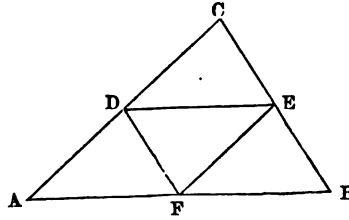


10. The straight line which joins the points of bisection of the sides of a triangle is parallel to the base.

Prove that the triangles  $ADB$  and  $AEB$  are each half of the given triangle by I. 38.; and therefore equal to each other, and therefore that  $DE$  is parallel to  $AB$ , by I. 39.

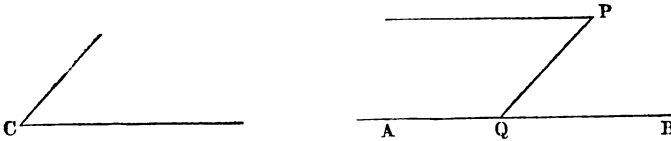


11. If the three points of bisection of the three sides of BOOK I  
a triangle be joined by  
straight lines, the triangle  
so formed shall have its  
angles equal to those of the  
original triangle.



Prove, as in the last, the  
sides to be respectively par-  
allel to the sides of the triangle, and then use I. 34.

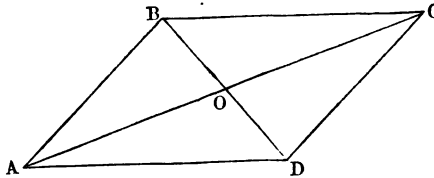
12. From a given point to draw a straight line, which shall  
make with a given straight line an angle equal to a given  
rectilineal angle.



Use in the construction I. 31. and I. 23.; and in the  
proof, I. 29.

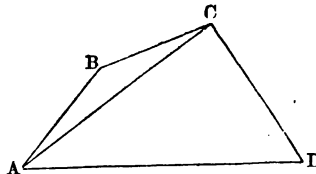
13. The diameters of  
a parallelogram bi-  
sect each other.

Use I. 34. I. 29.,  
and I. 26.



14. Any one side of a polygon  
is less than the sum of the  
remaining sides.

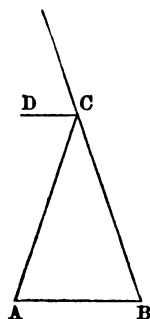
This follows from I. 20.



BOOK I.

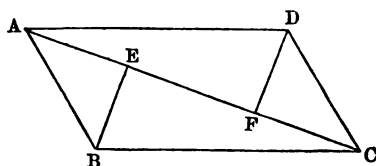
15. The straight line which bisects the exterior angle at the vertex of an isosceles triangle, is parallel to the base.

Use I. 32. and Def. 7.



16. The perpendiculars drawn from the angles of a parallelogram upon its diameter, are equal to each other.

Use I. 32., I. 34., and I. 26.



17. From a given point in the side of a triangle, to draw a straight line which shall divide it into two equal parts.

Let P be the point.

Bisect AB in O.

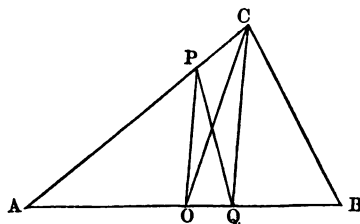
Join PO and CO.

Draw CQ parallel to PO.

Join PQ, which shall be the required line.

Show that the triangles POQ and CPO are equal.

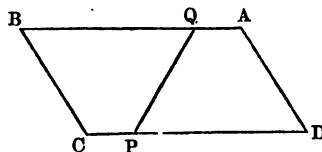
Add to each APO. Then ACO equals APQ, which, therefore, is equal to half the original triangle.



18. To bisect a parallelogram by a straight line drawn from a given point in one of its sides.

Cut off from AB, AQ equal to CP.

The straight line joining P and Q divides the parallelogram as required.

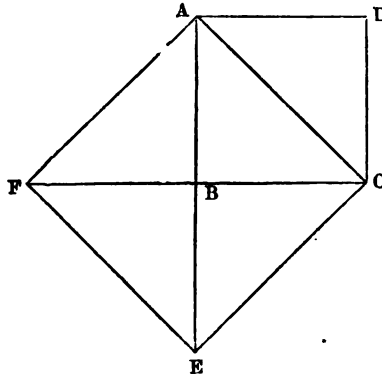


The proof of this is easy, and the learner may find it out BOOK I for himself.

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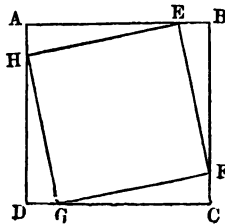
19. The square described upon the diagonal of a given square, is double of that square.

Prove the four triangles into which the larger square is divided by joining B and its angular points, to be each equal to either of the two triangles into which the original square is divided.



20. If four points be taken in the four sides of a square, at equal distances from the angular points respectively, the straight line which joins them will form a square.

Prove HEF to be a right angle, and then two adjacent sides of the figure, as HE, FE, to be equal.



## BOOK II.

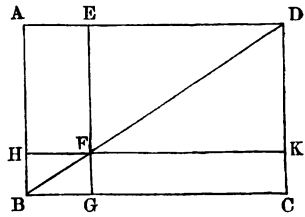
### DEFINITIONS.

#### I.

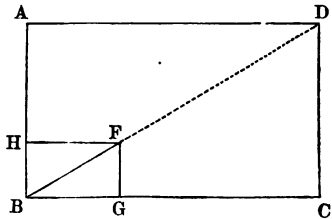
BOOK II. *Every right angled parallelogram is said to be contained by any two of the straight lines which contain one of the right angles.*

#### II.

In every parallelogram, any of the parallelograms about a diameter, together with the two complements, is called a Gnomon. "Thus the parallelogram HG, together with the complements AF, FC, is the gnomon, which is more briefly expressed by the letters AGK, or EHC, which are at the opposite angles of the parallelograms which make the gnomon."



[Observe that the parallelogram EK, together with the complements AF and FC is also a gnomon, and would be expressed by the letters AKG or CEH. In fact, a gnomon is the whole parallelogram, with the exception of one of the parallelograms about the diameter.]

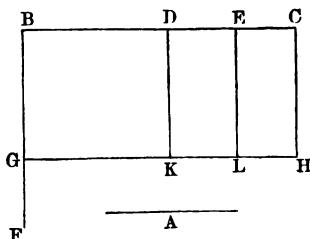


## PROP. I. THEOR.

*If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the rectangles contained by the undivided line, and the several parts of the divided line.*

Let A and BC be two straight lines ;  
and let BC be divided into any parts in the points D, E ;  
the rectangle contained by the straight lines A, BC  
is equal to the rectangle

contained by A, BD, together  
with that contained by A,  
DE, and that contained by  
A, EC.



From the point B draw<sup>a</sup> BF ;  
at right angles to BC,

and make BG equal<sup>b</sup> to A ;

and through G draw<sup>c</sup> GH parallel to BC ;

and through D, E, C, draw<sup>c</sup> DK, EL, CH, parallel to BG ;

<sup>a</sup> xi. 1.

<sup>b</sup> iii. 1.

<sup>c</sup> xxxi. 1.

Then the rectangle BH is equal to

the rectangles BK, DL, EH ;

and BH is contained by A, BC,

for it is contained by GB, BC,

and GB is equal to A ;

and BK is contained by A, BD,

for it is contained by GB, BD,

of which GB is equal to A ;

and DL is contained by A, DE,

because DK, that is<sup>d</sup> BG, is equal to A ;

<sup>d</sup> xxxiv. 1.

and in like manner the rectangle EH is contained by A, EC :

Therefore the rectangle contained by A, BC is equal to

the several rectangles contained by A, BD, and by A, DE ;

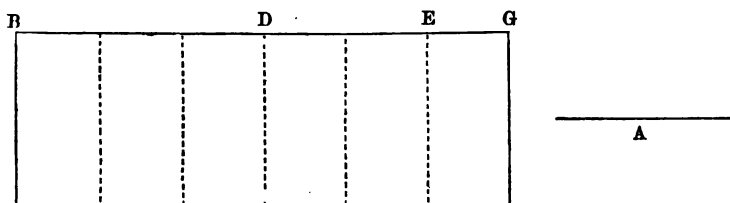
and also by A, EC.

Wherefore, if there be two straight lines, &c. Q. E. D.

## BOOK II

[This may be illustrated by measuring six equal distances from B to G, and terminating the straight line at the end of the sixth, so that BD shall contain three of the distances, DE two, and EG one, and by taking the other straight line A of any length.

Through each of the points of the divided line draw lines



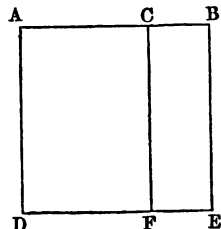
parallel to B, and the figure will be divided into six equal parallelograms, which are together equal to the first three, and the next two, and the remaining one; or, in numbers, six is equal to three, and two and one added together.]

## PROP. II. THEOR.

*If a straight line be divided into any two parts, the rectangles contained by the whole and each of the parts are together equal to the square of the whole line.*

Let the straight line AB be divided into any two parts in the point C;

the rectangle contained by AB, BC;  
together with the rectangle\* AB, AC,  
shall be equal to the square of AB.



\* xlv. 1.

Upon AB describe\* the square ADEB,

<sup>b</sup> xxxi. 1.

and through C draw<sup>b</sup> CF, parallel to AD or BE;

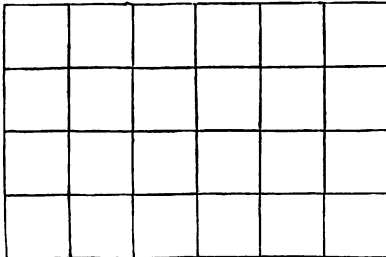
Then AE is equal to the rectangles AF, CE;

\* N.B. To avoid repeating the word *contained* too frequently, the rectangle contained by two straight lines, AB, AC, is sometimes simply called the rectangle AB, AC.

and AE is the square of AB ;  
 and AF is the rectangle contained by BA, AC ;  
 for it is contained by DA, AC, of which AD is equal to AB ;  
 and CE is contained by AB, BC,  
 for BE is equal to AB ;  
 therefore the rectangle contained by AB, AC,  
 together with the rectangle AB, BC,  
 is equal to the square of AB.  
 If therefore a straight line, &c. Q. E. D.

[Let the straight line AB consist of five equal parts, of which AC shall contain three and CB two.

By dividing the straight line AD into five equal parts, it will be seen that the whole square contains five times five, or twenty-five equal squares. The parallelogram AF containing fifteen of them, and CE ten, which numbers added together make twenty-five. It should be observed that if one side of a rectangle contains any given number of units (six for instance), and the other side any other given number (four for instance), the whole rectangle will contain the number of squares (in this case twenty-four) which is equal to the two numbers multiplied together.]



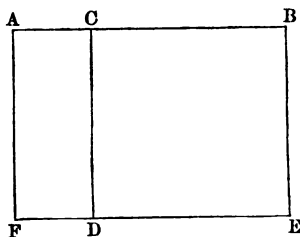
## BOOK II

## PROP. III. THEOR.

*If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square of the aforesaid part.*

Let the straight line AB be divided into two parts in the point C:  
the rectangle AB, BC is equal to  
the rectangle AC, CB, together with the square of BC.

- <sup>a</sup> xli. 1. Upon BC describe<sup>a</sup> the square CDEB,  
and produce ED to F,  
<sup>b</sup> xxxi. 1. and through A draw<sup>b</sup> AF parallel to CD or BE;



Then the rectangle AE is equal  
to the rectangles AD, CE;  
and AE is the rectangle contained by AB, BE,  
for it is contained by AB, BE,  
of which BE is equal to BC;  
and AD is contained by AC, CB,  
for CD is equal to BC;  
and DB is the square of BC;  
therefore the rectangle AB, BC  
is equal to the rectangle AC, CB, together with the square  
of BC.

If therefore a straight line, &c. Q. E. D.

[This may be illustrated in the same way as the two preceding Propositions numerically, thus:—Let the whole line contain 12 units, and be divided into 7 and 5. Then

12 multiplied by 5 is equal to 60.  
7 multiplied by 5 is equal to 35,  
5 (squared or) multiplied by 5 is equal to 25.  
These two added together make up 60.

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Or, again,  
12 multiplied by 7 is equal to 84.  
7 multiplied by 5 is equal to 35,  
7 (squared or) multiplied by 7 is equal to 49.  
These added together make up 84.]

## BOOK II

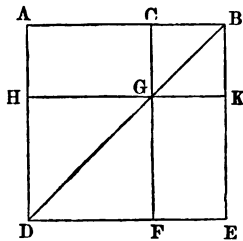
## PROP. IV. THEOR.

*If a straight line be divided into any two parts, the square of the whole line is equal to the squares of the two parts, together with twice the rectangle contained by the parts.*

Let the straight line AB be divided into any two parts in C;  
the square of AB is equal to  
the squares of AC, CB,  
and to twice the rectangle contained by AC, CB.

- \* xlv. 1. Upon AB describe<sup>a</sup> the square ADEB,  
and join BD,  
\* xxxi. 1. and through C draw<sup>b</sup> CGF parallel to AD or BE,  
and through G draw HK parallel to AB or DE.

- And because CF is parallel to AD, and BD falls upon them,  
\* xxix. 1. the exterior angle BGC is equal<sup>c</sup> to  
the interior and opposite angle ADB;  
\* v. 1. but ADB is equal<sup>d</sup> to the angle ABD,  
because BA is equal to AD,  
being sides of a square;  
wherefore the angle CGB is equal to  
the angle GBC;  
\* vi. 1. and therefore the side BC is equal<sup>e</sup> to  
the side CG:  
\* xxxiv. 1. But CB is equal<sup>f</sup> also to GK,  
and CG to BK;  
wherefore the figure CGKB is equi-  
lateral:



It is likewise rectangular;  
for CG is parallel to BK, and CB meets them;  
the angles KBC, GCB are therefore equal to two right angles,  
and KBC is a right angle;  
wherefore GCB is a right angle;  
and therefore also the angles<sup>f</sup> CGK, GKB, opposite to these,  
are right angles,

and  $CGKB$  is rectangular ;  
 but it is also equilateral, as was demonstrated ;  
 wherefore it is a square, and it is upon the side  $CB$  :  
 For the same reason  $HF$  also is a square,  
 and it is upon the side  $HG$ , which is equal to  $AC$  :  
 Therefore  $HF$ ,  $CK$  are the squares of  $AC$ ,  $CB$  ;  
 and because the complement  $AG$   
 is equal<sup>e</sup> to the complement  $GE$ ,  
 and that  $AG$  is the rectangle contained by  $AC$ ,  $CB$ ,  
 for  $GC$  is equal to  $CB$  ;  
 therefore  $GE$  is also equal to the rectangle  $AC$ ,  $CB$  ;  
 wherefore  $AG$ ,  $GE$  are equal to twice the rectangle  $AC$ ,  $CB$  :  
 And  $HF$ ,  $CK$ , are the squares of  $AC$ ,  $CB$  ;  
 wherefore the four figures  $HF$ ,  $CK$ ,  $AG$ ,  $GE$ , are equal to  
 the squares of  $AC$ ,  $CB$ , and to twice the rectangle  $AC$ ,  $CB$  :  
 But  $HF$ ,  $CK$ ,  $AG$ ,  $GE$ , make up the whole figure  $ADEB$ ,  
 which is the square of  $AB$  :  
 Therefore the square of  $AB$  is equal to  
 the squares of  $AC$ ,  $CB$ , and twice the rectangle  $AC$ ,  $CB$ .  
 Wherefore if a straight line, &c. Q. E. D.

\* xliii. 1.

COR. From the demonstration it is manifest, that  
 the parallelograms about the diameter of a square  
 are likewise squares.

[Take the number 8, and divide it into two parts, 5 and 3.  
 The square of the whole number is equal to 64.  
 The square of 5 is equal to 25.  
 The square of 3 is equal to 9.  
 Twice the product of 5 and 3 is 30, for their product is 15.  
 Adding these together the sum is 64.

The same may be shown of any other division of 8, as into  
 6 and 2, or 7 and 1.]

## BOOK II.

## PROP. V. THEOR.

*If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square of half the line.*

Let the straight line AB be divided  
into two equal parts in the point C,  
and into two unequal parts at the point D;  
the rectangle AD, DB, together with the square of CD,  
is equal to the square of CB.

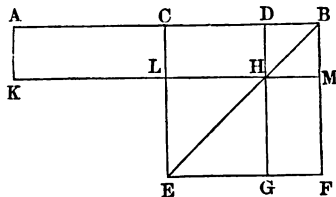
- <sup>a</sup> xlii. 1. Upon CB describe<sup>a</sup> the square CEFB,  
join BE,  
<sup>b</sup> xxxi. 1. and through D draw<sup>b</sup> DHG parallel to CE or BF;  
and through H draw KLM parallel to CB or EF;  
and also through A draw AK parallel to CL or BM:

- And because the complement CH  
<sup>c</sup> xliii. 1. is equal<sup>c</sup> to the complement HF,  
to each of these add DM;  
therefore the whole CM is equal to the whole DF;  
<sup>d</sup> xxxvi. 1. but CM is equal<sup>d</sup> to AL;  
because AC is equal to CB;  
therefore also AL is equal to DF.

To each of these add CH,  
and the whole AH is equal  
to DF and CH:

But AH is the rectangle  
contained by AD, DB,

- <sup>e</sup> Cor. iv. 2. for DH is equal<sup>e</sup> to DB;  
and DF together with CH  
is the gnomon CMG;  
therefore the gnomon CMG  
is equal to the rectangle AD, DB:  
To each of these add LG,



which is equal<sup>e</sup> to the square of CD;

therefore the gnomon CMG, together with LG, is equal to the rectangle AD, DB, together with the square of CD; • Cor. iv. 2.

But the gnomon CMG and LG make up the whole figure CEFB,

which is the square of CB:

Therefore the rectangle AD, DB,

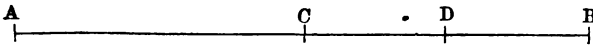
together with the square of CD,

is equal to the square of CB.

Wherefore if a straight line, &c. Q. E. D.

From this Proposition it is manifest, that the difference of the squares of two unequal lines AC, CD, is equal to the rectangle contained by their sum and difference.

[For suppose AD to represent the sum of two unequal lines, and DB to represent their difference.



Bisect AB in C. Then CB and CD shall represent the lines themselves, for AD is equal to AC and CD together, which is equal to CB and CD together; also DB is manifestly the difference between CB and CD.

And now we have exactly the case of the Proposition just demonstrated, viz.

The rectangle AD, DB, together with the square of CD, is equal to the square of CB.

Take away from these equals the square of CD, and we have the rectangle AD, DB equal to the square of CB, with the square of CD taken from it; or equal to the difference of the squares of CB and CD, or the difference of the squares of two unequal lines, AC, CD, is equal to the rectangle contained by their sum and difference.]

## BOOK II

## PROP. VI. THEOR.

*If a straight line be bisected, and produced to any point: the rectangle contained by the whole line thus produced, and the part of it produced, together with the square of half the line bisected, is equal to the square of the straight line which is made up of the half and the part produced.*

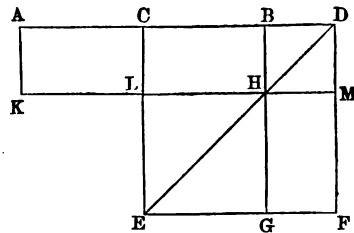
Let the straight line AB be bisected in C,  
and produced to the point D;  
the rectangle AD, DB, together with the square of CB,  
is equal to the square of CD.

- <sup>a</sup> xlvi. 1. Upon CD describe<sup>a</sup> the square CEFD,  
join DE,  
<sup>b</sup> xxxi. 1. and through B draw<sup>b</sup> BHG parallel to CE or DF,  
and through H draw KLM parallel to AD or EF,  
and also through A draw AK parallel to CL or DM,

And because AC is equal to  
CB,

- <sup>c</sup> xxxvi. 1. the rectangle AL is equal<sup>c</sup>  
to CH;

- <sup>d</sup> xliii. 1. but CH is equal<sup>d</sup> to HF;  
therefore also AL is equal  
to HF:



To each of these add CM;  
therefore the whole AM is equal to the gnomon CMG:  
And AM is the rectangle contained by AD, DB,

- <sup>e</sup> Cor iv. 2. for DM is equal<sup>e</sup> to DB.

Therefore the gnomon CMG is equal to  
the rectangle AD, DB:  
Add to each of these LG,  
which is equal to the square of CB,  
therefore the rectangle AD, DB,  
together with the square of CB,  
is equal to the gnomon CMG, and the figure LG;

But the gnomon CMG and LG  
make up the whole figure CEFD,  
which is the square of CD;  
therefore the rectangle AD, DB,  
together with the square of CB,  
is equal to the square of CD.

Wherefore, if a straight line, &c. Q. E. D.

PROP. VII. THEOR.

*If a straight line be divided into any two parts, the squares of the whole line, and of one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square of the other part.*

Let the straight line AB be divided into any two parts in the point C;  
the squares of AB, BC are equal to  
twice the rectangle AB, BC, together with the square of AC.

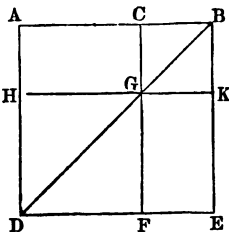
Upon AB describe <sup>a</sup> the square ADEB,  
and construct the figure as in the preceding Propositions;

<sup>a</sup> xlvi. 1.

And because AG is equal <sup>b</sup> to GE,  
add to each of them CK;  
the whole AK is therefore equal to the whole CE;  
therefore AK, CE are double of AK:  
But AK, CE are the gnomon AKF,  
together with the square CK;  
therefore the gnomon AKF, together  
with the square CK, is double of  
AK:

<sup>b</sup> xliii. 1.

But twice the rectangle AB, BC is  
double of AK,



for BK is equal <sup>c</sup> to BC:

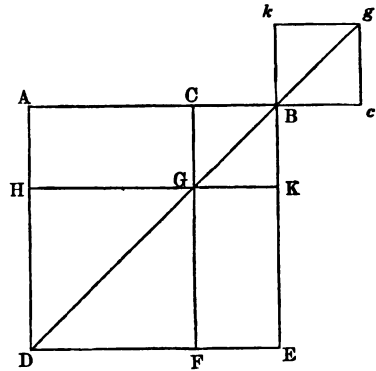
<sup>c</sup> Cor. iv. 2.

Therefore the gnomon AKF, together with the square CK,  
is equal to twice the rectangle AB, BC:

To each of these equals add HF,

BOOK II which is equal to the square of AC;  
 therefore the gnomon AKF,  
 together with the squares CK, HF,  
 is equal to twice the rectangle AB, BC,  
 and the square of AC;  
 but the gnomon AKF, together with the squares CK, HF,  
 make up the whole figure ADEB and CK,  
 which are the squares of AB and BC :  
 therefore the squares of AB and BC are equal to  
 twice the rectangle AB, BC,  
 together with the square of AC.  
 Wherefore, if a straight line, &c. Q. E. D.

[In proving this Proposition, the work of constructing the figure as in the preceding Proposition should be gone through, either *vivâ voce*, or on paper, as the case may be, and the figure should be actually drawn again though it is only a repetition of the figure of Prop. IV. It may be worth while to observe, that if the two sides AB, EB are produced to *c* and *k*, BK being equal to B*k*, and B*c* to BC, the two things proved equal to each other in this Proposition are equal to the two squares AE and *k**c* added together.]



## PROP. VIII. THEOR.

*If a straight line be divided into any two parts, four times the rectangle contained by the whole line and one of the parts, together with the square of the other part, is equal to the square of the straight line, which is made up of the whole and that part.*

[This Proposition may advantageously be omitted at the first time of reading. It should be afterwards read, though it is of very little use.]

Let the straight line AB be divided into any two parts in the point C; four times the rectangle AB, BC, together with the square of AC, is equal to the square of the straight line made up of AB and BC together.

Produce AB to D, so that BD be equal to CB, and upon AD describe the square AEFD; and construct two figures such as in the preceding.

Because CB is equal to BD,  
and that CB is equal <sup>a</sup> to GK,  
and BD to KN;

<sup>a</sup> xxxiv. 1.

therefore GK is equal to KN:  
For the same reason, PR is equal to RO;  
and because CB is equal to BD,

and GK to KN,  
the rectangle CK is equal <sup>b</sup> to BN,  
and GR to RN;

<sup>b</sup> xxxvi. 1.

but CK is equal <sup>c</sup> to RN,  
because they are the complements of the parallelogram CO;  
therefore also BN is equal to GR;  
and the four rectangles BN, CK, GR, RN  
are therefore equal to another,

<sup>c</sup> xliii. 1.

and so are quadruple of one of them CK:  
Again, because CB is equal to BD,  
and that BD is equal <sup>d</sup> to BK,

<sup>d</sup> Cor. iv. 2.

BOOK II that is, to CG,

and CB equal to GK,

<sup>a</sup> Cor. iv. 2. that <sup>d</sup> is, to GP;

therefore CG is equal to GP;

And because CG is equal to GP,

and PR to RO,

the rectangle AG is equal to MP,

and PL to RF:

• xliii. 1. But MP is equal <sup>e</sup> to PL,

because they are the complements

of the parallelogram ML;

wherefore AG is equal also to RF:

Therefore the four rectangles AG, MP, PL, RF,

are equal to one another,

and so are quadruple of one of them AG.

And it was demonstrated that the four CK, BN, GR, RN,

are quadruple of CK.

Therefore the eight rectangles which contain the gnomon

AOH, are quadruple of AK;

and because AK is the rectangle contained by AB, BC,

for BK is equal to BC,

four times the rectangle AB, BC is quadruple of AK:

But the gnomon AOH was demonstrated to be quadruple of AK;

therefore four times the rectangle AB, BC,

is equal to the gnomon AOH.

To each of these add XH,

<sup>f</sup> Cor. iv. 2. which is equal <sup>f</sup> to the square of AC:

Therefore four times the rectangle AB, BC,

together with the square of AC,

is equal to the gnomon AOH and the square XH:

But the gnomon AOH and XH make up the figure AEFD,

which is the square of AD:

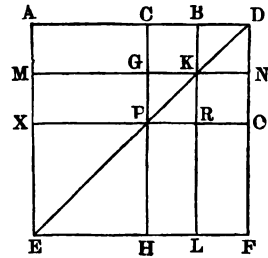
Therefore four times the rectangle AB, BC,

together with the square of AC,

is equal to the square of AD,

that is, of AB and BC added together in one straight line.

Wherefore, if a straight line, &c. Q. E. D.



## PROP. IX. THEOR.

*If a straight line be divided into two equal, and also into two unequal parts; the squares of the two unequal parts are together double of the square of half the line, and of the square of the line between the points of section.*

Let the straight line AB be divided  
at the point C into two equal,  
and at D into two unequal parts:  
the squares of AD, DB are together double of  
the squares of AC, CD.

From the point C draw <sup>a</sup> CE at right angles to AB,  
and make it equal to AC or CB,  
and join EA, EB;  
through D draw <sup>b</sup> DF parallel to CE,  
and through F draw FG parallel to AB,  
and join AF:

• xi. 1.

• xxxi. 1.

Then, because AC is equal to CE,  
the angle EAC is equal <sup>c</sup> to the angle AEC;  
and because the angle ACE is a right angle,  
the two others AEC, EAC together make one right angle <sup>d</sup>; <sup>e</sup> xxxii. 1.  
and they are equal to one  
another;

each of them therefore is half  
of a right angle.

For the same reason, each of  
the angles CEB, EBC

is half a right angle;

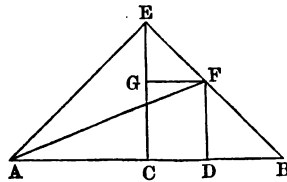
and therefore the whole AEB is a right angle:

And because the angle GEF is half a right angle,

and EGF a right angle,

for it is equal <sup>e</sup> to the interior and opposite angle ECB,  
the remaining angle EFG is half a right angle;

• xxix. 1.

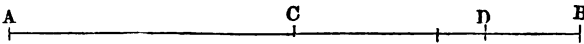


- BOOK II therefore the angle GEF is equal to the angle EFG,  
 and the side EG equal <sup>r</sup> to the side GF:  
 Again, because the angle at B is half a right angle,  
 and FDB a right angle,  
 • xxix. 1. for it is equal <sup>e</sup> to the interior and opposite angle ECB,  
 the remaining angle BFD is half a right angle;  
 therefore the angle at B is equal to the angle BFD,  
 and the side DF to <sup>r</sup> the side DB:  
 And because AC is equal to CE,  
 the square of AC is equal to the square of CE;  
 therefore the squares of AC, CE are double of the square  
 of AC:  
 • xlvii. 1. But the square of EA is equal <sup>s</sup> to the squares of AC, CE,  
 because ACE is a right angle;  
 therefore the square of EA is double of the square of AC:  
 Again, because EG is equal to GF,  
 the square of EG is equal to the square of GF;  
 therefore the squares of EG, GF are double of the square of  
 GF;  
 but the square of EF is equal to the squares of EG, GF;  
 therefore the square of EF is double of the square GF;  
 • xxxiv. 1. and GF is equal <sup>h</sup> to CD;  
 therefore the square of EF is double of the square of CD:  
 But the square of AE is likewise double of the square of  
 AC;  
 therefore the squares of AE, EF are double of the squares  
 of AC, CD:  
 • xlvii. 1. And the square of AF is equal <sup>i</sup> to the squares of AE, EF,  
 because AEF is a right angle;  
 therefore the square of AF is double of the squares of AC,  
 CD:  
 But the squares of AD, DF are equal to the square of AF,  
 because the angle ADF is a right angle:  
 therefore the squares of AD, DF are double of the squares  
 of AC, CD:  
 And DF is equal to DB;

therefore the squares of AD, DB are double of the squares of BOOK II.  
AC, CD.

If therefore a straight line, &c. Q. E. D.

[The demonstration of this Proposition will be more easily remembered, if it is borne in mind that the two things to be proved equal to each other are so proved by showing that they are both equal to the square of AF.]



Suppose AB equal to 12,

AD equal to 10 :

Then CD equals 4,

DB equals 2.

The squares of the two unequal parts are 100 and 4, which added together are double of the square of half the line (36), and the square of the line between the points of section (16), added together, *i. e.* double of 52.]

## BOOK II.

## PROP. X. THEOR.

*If a straight line be bisected, and produced to any point, the square of the whole line thus produced, and the square of the part of it produced, are together double of the square of half the line bisected, and of the square of the line made up of the half and the part produced.*

Let the straight line AB be bisected in C,  
and produced to the point D ;  
the squares of AD, DB are double of the squares of AC, CD.

\* xi. 1. From the point C draw <sup>a</sup> CE at right angles to AB :  
And make it equal to AC or CB,  
and join AE, EB ;

• xxxi. 1. through E draw <sup>b</sup> EF parallel to AB,  
and through D draw DF parallel to CE :

And because the straight line EF meets the parallels EC, FD,  
• xxix. 1. the angles CEF, EFD are equal <sup>c</sup> to two right angles,  
and therefore the angles BEF, EFD are less than two right  
angles ;

but straight lines which with another straight line make the  
interior angles upon the same side less than two right  
angles, do meet <sup>d</sup> if produced far enough :

<sup>d</sup> 12 Ax. Therefore EB, FD shall meet, if produced towards B, D :  
Let them meet in G,  
and join AG :

Then, because AC is equal to CE,  
• v. 1. the angle CEA is equal <sup>e</sup> to the angle EAC :  
and the angle ACE is a right angle ;

• xxii. 1. therefore each of the angles CEA, EAC is half a right angle.<sup>f</sup>  
For the same reason,  
each of the angles CEB, EBC is half a right angle ;  
therefore AEB is a right angle :

And because EBC is half a right angle,  
• vi. 1. DBG is also <sup>g</sup> half a right angle,

for they are vertically opposite ;

but BDG is a right angle,

because it is equal <sup>a</sup> to the alternate angle DCE ;

therefore the remaining angle DGB is half a right angle,

and is therefore equal to the angle DBG ;

wherefore also the side BD is equal <sup>b</sup> to the side DG :

Again, because EGF is half a right angle,

and that the angle at F is

a right angle,

because it is equal <sup>b</sup> to the

opposite angle ECD,

the remaining angle FEG

is half a right angle,

and equal to the angle

EGF ;

wherefore also the side GF is equal <sup>a</sup> to the side FE.

And because EC is equal to CA,

the square of EC is equal to the square of CA ;

therefore the squares of EC, CA are double of the square of

CA.

But the square of EA is equal <sup>1</sup> to the squares of EC, CA ; <sup>1</sup> xlvii. 1.

therefore the square of EA is double of the square of AC :

Again, because GF is equal to FE,

the square of GF is equal to the square of FE :

and therefore the squares of GF, FE are double of the square of EF :

But the square of EG is equal <sup>1</sup> to the squares of GF, FE ;

therefore the square of EG is double of the square of EF :

And EF is equal to CD ;

wherefore the square of EG is double of the square of CD.

But it was demonstrated, that

the square of EA is double of the squares of AC ;

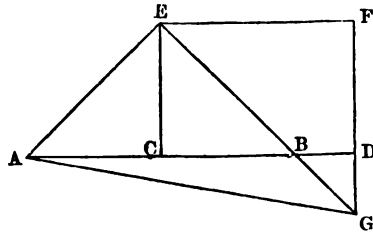
therefore the squares of AE, EG,

are double of the squares of AC, CD ;

And the square of AG is equal <sup>1</sup> to the squares of AE, EG ;

therefore the square of AG is double of the square of AC,

CD :



<sup>a</sup> xxix. 1.

<sup>b</sup> vi. 1.

<sup>b</sup> xxxiv. 1.

<sup>1</sup> xlvii. 1.

BOOK II. But the squares of AD, GD are equal<sup>1</sup> to the square of AG ;

<sup>1</sup> xlvii. 1. therefore the squares of AD, DG are double of  
the squares of AC, CD :

But DG is equal to DB ;

therefore the squares of AD, DB are double of  
the squares of AC, CD.

Wherefore, if a straight line, &c. Q. E. D.

[In this case also it is to be remembered, that the two things to be proved equal to each other, are so proved by showing that they are each equal to the square of AG.

Suppose AB equal to 6,

BD equal to 2 :

Then AC equals 3,

and CD equals 5.

The square of the whole line produced (64), and the square of the parts produced (4), are together double of the square of half the line bisected (9), and the square of the line made up of the half and part produced (25).

These two Propositions may be omitted at the first reading.]

## PROP. XI. PROB.

*To divide a given straight line into two parts, so that the rectangle contained by the whole, and one of the parts, shall be equal to the square of the other part.*

Let AB be the given straight line ;  
it is required to divide it into two parts, so that  
the rectangle contained by the whole, and one of the parts,  
shall be equal to the square of the other part.

Upon AB describe<sup>a</sup> the square ABDC ;  
bisect<sup>b</sup> AC in E,  
and join BE ;  
produce CA to F,  
and make<sup>c</sup> EF equal to EB,  
and upon AF describe<sup>a</sup> the square of FGHA ;  
AB is divided in H, so that the rectangle AB, BH,  
is equal to the square of AH.

<sup>a</sup> xlv. 1.<sup>b</sup> x. 1.<sup>c</sup> iii. 1.

Produce GH to K ;

Because the straight line AC is bisected in E,  
and produced to the point F,  
the rectangle CF, FA, together with the square of AE,  
is equal<sup>d</sup> to the square of EF :

<sup>d</sup> v. 2.

But EF is equal to EB ;  
therefore the rectangle CF, FA, together with the square of  
AE,

is equal to the square of EB ;  
and the squares of BA, AE, are equal<sup>e</sup> to the square of EB, <sup>e</sup> xlvii. 1.  
because the angle EAB is a right angle ;  
therefore the rectangle CF, FA, together with the square  
of AE,

is equal to the squares of BA, AE :

Take away the square of AE, which is common to both,  
therefore the remaining rectangle CF, FA,  
is equal to the square of AB ;

BOOK II. and the figure FK is the rectangle  
contained by CF, FA,

for AF is equal to FG:

and AD is the square of AB;

therefore FK is equal to AD:

Take away the common part AK,

and the remainder FH is equal to the  
remainder HD:

And HD is the rectangle contained  
by AB, BH,

for AB is equal to BD;

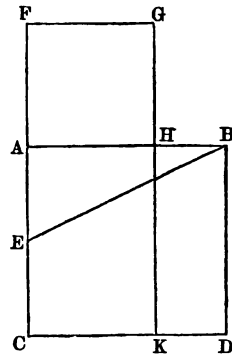
and FH is the square of AH.

Therefore the rectangle AB, BH is equal to the square of  
AH:

Wherefore, the straight line AB is divided in H,

so that the rectangle AB, BH, is equal to the square of AH.

Which was to be done.



[It is impossible to exhibit this Proposition in exact numbers. The reader may try with any given number, *e. g.* 12., and will find that it cannot be done.

Thus, if 7 and 5 be taken for the two parts,

12 multiplied by 5 equals 60, which is larger than the square  
of 7 or 49.

But if 8 and 4 be tried,

12 multiplied by 4 equals 48, which is less than 64, or the  
square of 8.]

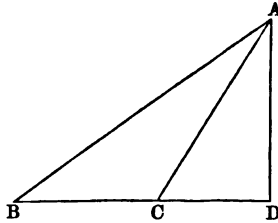
## PROP. XII. THEOR.

*In obtuse angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square of the sides subtending the obtuse angle is greater than the squares of the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and the obtuse angle.*

Let ABC be an obtuse angled triangle,  
having the obtuse angle ACB,  
and from the point A let AD be drawn <sup>a</sup>  
perpendicular to BC produced:  
The square of AB is greater than the squares of AC, CB,  
by twice the rectangle BC, CD.

<sup>a</sup> xii. 1.

Because the straight line BD is divided into two parts in  
the point C,  
the square of BD is equal <sup>b</sup> to  
the squares of BC, CD,  
and twice the rectangle BC, CD;  
To each of these equals add the  
square of DA,  
and the squares of BD, DA are  
equal to  
the squares of BC, CD, DA, and twice the rectangle BC,  
CD:



<sup>b</sup> iv. 2.

But the square of BA is equal <sup>c</sup> to the squares of BD, DA, <sup>c</sup> xlvii. 1.  
because the angle at D is a right angle:  
[Therefore the square of BA is equal to the squares of BC,  
CD, DA, and twice the rectangle BC, CD:]  
and the square of CA is equal <sup>c</sup> to the squares of CD, DA:  
Therefore the square of BA is equal to

**BOOK II.** the squares of  $BC$ ,  $CA$ , and twice the rectangle  $BC$ ,  $CD$ ;  
 that is, the square of  $BA$  is greater than the squares of  $BC$ ,  
 $CA$ ,

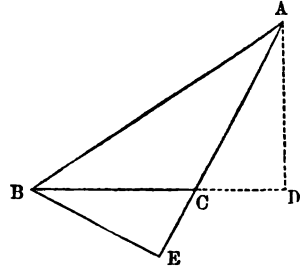
by twice the rectangle  $BC$ ,  $CD$ .

Therefore, in obtuse angled triangles, &c. Q. E. D.

[Before going on to the next Proposition, the reader should satisfy himself that he understands this by demonstrating it again in the case of  $AC$  being produced.

In this figure the square of  $AB$  is greater than the square of  $AC$ ,  $CB$ , by twice the rectangle  $AC$ ,  $CE$ .

**COR.** From this it appears, that the rectangle  $AC$ ,  $CE$  is equal to the rectangle  $BC$ ,  $CD$ .]

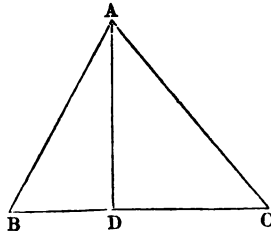


## PROP. XIII. THEOR.

*In every triangle, the square of the side subtending any of the acute angles is less than the squares of the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall upon it from the opposite angle, and the acute angle.*

Let ABC be any triangle,  
and the angle at B one of its acute angles,  
and upon BC, one of the sides containing it,  
let fall the perpendicular <sup>a</sup> AD from the opposite angle: <sup>a</sup> xii. 1.  
The square of AC, opposite to the angle B,  
is less than the squares of CB, BA, by twice the rectangle  
CB, BD.

CASE I. First, let AD fall within the triangle ABC;  
and because the straight line CB  
is divided into two parts in the  
point D,  
the squares of CB, BD are equal <sup>b</sup>  
to twice the rectangle contained  
by CB, BD, and the square of  
DC:



<sup>b</sup> vii. 2.

To each of these equals add the  
square of AD;  
therefore the squares of CB, BD, DA are equal to  
twice the rectangle CB, BD; and the squares of AD, DC:  
But the square of AB is equal <sup>c</sup> to the squares of BD, DA, <sup>c</sup> xlvii. 1.  
because the angle BDA is a right angle;  
[Therefore the squares of CB, BA are equal to  
twice the rectangle CB, BD; and the squares of AD, DC:]  
and the square of AC is equal to the squares of AD, DC:  
Therefore the squares of CB, BA are equal to  
the square of AC, and twice the rectangle CB, BD,  
that is,  
the square of AC alone is less than  
the squares of CB, BA by twice the rectangle CB, BD.

BOOK II. CASE II. Secondly, let AD fall

without the triangle ABC:

Then, because the angle at D is a right angle,

<sup>d</sup> xvi. 1. the angle ACB is greater<sup>d</sup> than a right angle;

and therefore the square of AB is equal<sup>e</sup> to

xii. 2.

the squares of AC, CB, and twice the rectangle BC, CD:

To these equals add the square of BC,

and the squares of AB, BC are equal to the square of AC,

and twice the square of BC, and twice the rectangle BC, CD:

But because BD is divided into two parts in C,

<sup>f</sup> iii. 2.

the rectangle DB, BC is equal<sup>f</sup> to

the rectangle BC, CD and the square of BC:

And the doubles of these are equal:

[that is, twice the rectangle DB, BC is equal to

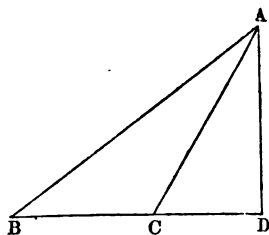
twice the rectangle BC, CD and twice the square of BC:]

Therefore the squares of AB, BC are equal to

the square of AC, and twice the rectangle DB, BC:

Therefore the square of AC alone is less than

the squares of AB, BC, by twice the rectangle DB, BC.



CASE III. Lastly, let the side AC be perpendicular to BC;

then is BC the straight line between the perpendicular and the acute angle at B;

and it is manifest, that the squares of AB,

<sup>g</sup> xlvii. 1.

BC are equal<sup>g</sup> to the square of AC and twice the square of BC:

Therefore, in every triangle, &c. Q. E. D.



[The last case, though necessary for the completion of the proof, is of no further use. It is only in reality an involved mode of stating, that the square on the side subtending the right angle is equal to the sum of the squares on the two other sides of a right angled triangle.]

## PROP. XIV. PROB.

## BOOK II

*To describe a square that shall be equal to a given rectilineal figure.*

Let A be the given rectilineal figure; it is required to describe a square that shall be equal to A.

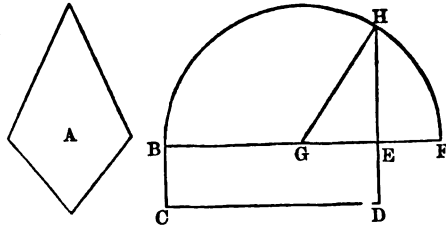
Describe <sup>a</sup> the rectangular parallelogram BCDE equal to the rectilineal figure A.

<sup>a</sup> xlv. 1.

If then the sides of it BE, ED are equal to one another, it is a square,

and what was required is now done:

But if they are not equal, produce one of them BE to F,



and make EF equal to ED

and bisect BF in G;

and from the centre G, at the distance GB, or GF, describe the semicircle BHF,

and produce DE to H,

and join GH:

Therefore because the straight line BF is divided into two equal parts in the point G, and into two unequal at E,

the rectangle BE, EF, together with the square of EG, is equal <sup>b</sup> to the square of GF:

<sup>b</sup> v. 2.

But GF is equal to GH:

therefore the rectangle BE, EF, together with the square of EG,

is equal to the square of GH:

But the squares of HE, EG are equal <sup>c</sup> to the square of <sup>d</sup> GH:

GH:

Therefore the rectangle BE, EF,

BOOK II. together with the square of EG,  
is equal to the squares of HE, EG:  
Take away the square of EG,  
which is common to both;  
and the remaining rectangle BE, EF is equal to the square  
of EH:  
But the rectangle contained by BE, EF is the parallelogram  
BD,  
because EF is equal to ED;  
therefore BD is equal to the square of EH;  
but BD is equal to the rectilineal figure A;  
therefore the rectilineal figure A is equal to the square of EH.  
Wherefore a square has been made equal to  
the given rectilineal figure A;  
viz. the square described upon EH.  
Which was to be done.

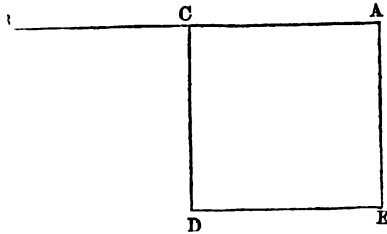
[In constructing this figure it will be necessary to divide the rectilineal figure A into triangles, and construct a rectangle equal to one of them, and then apply to one of the sides of that rectangle another rectangle which shall be equal to another of the triangles, and so on till a rectangle has been made equal to the whole rectilineal figure. Then proceed as directed in the Proposition. The learner should here draw correctly the figure which he was directed to omit at the first reading of i. 45.]

DEDUCTIONS FROM BOOK II.

THERE is not much of interest or importance in the Deductions usually appended to this book. The learner may exercise himself with the following, or proceed immediately to the Third Book.

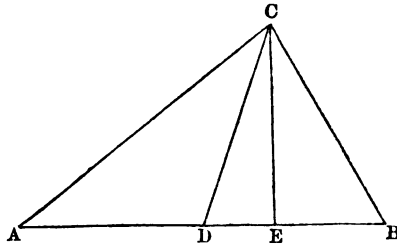
- (1.) To divide a straight line so that the rectangle of the parts shall be equal to the square of half the line.

This is an easy deduction from Prop. V., or may be done independently of it.



- (2.) If a straight line be drawn from the vertical angle of a triangle bisecting the base, the squares of the two sides of the triangle are together double of the squares of the bisecting line, and of half the base.

The square of AC is greater than the squares of AD, DC, by twice the rectangle AD, DE (II. 12.).



Or, in other words, is equal to the squares of AD, DC, together with twice the rectangle AD, DE; that is, is equal to the squares of AD, DC, together with twice the rectangle BD, DE, for AD is equal to BD.

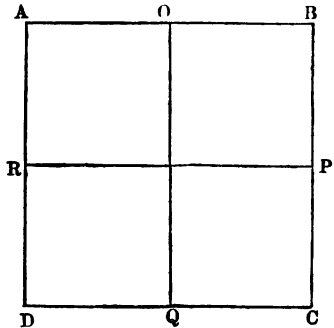
BOOK II. Again :

The square of BC is equal to the squares of BD, DC,  
less twice the rectangle BD, DE (II. 13.).

Add these equals together, and the Proposition is proved.

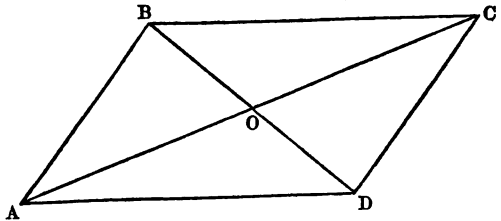
- (3.) The square described upon the whole line is equal to four times the square described upon half the line.

Prove OP, PQ, QR, RO, to be equal squares.



- (4.) The squares described upon the sides of a parallelogram are together equal to the squares upon its diameters.

Remember that the diagonals of a parallelogram bisect each other.

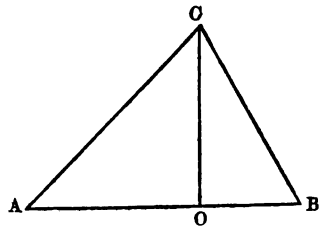


Then apply Ex. 2. to both the triangles ABC, ADC.

Then use Ex. 3.

- (5.) In any triangle, if a perpendicular be drawn from one of the angles to the opposite side, the difference of the squares described upon the sides is equal to the difference of the squares on the segment of the base.

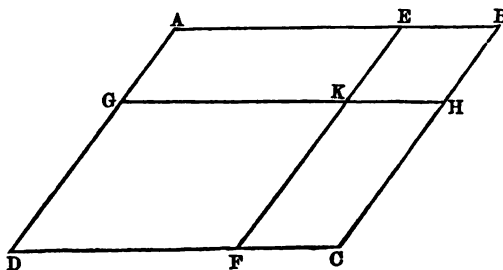
The square on AC is equal to the squares on AO, OC.



The square on BC is equal to the squares on BO, OC.  
 Subtract these equals from each other.

BOOK II.

- (6.) Prove that if a straight line be divided into two parts, the rhombus described upon the whole line shall be equal to those which are described upon the parts, and have their angles equal to its angles, together with twice the parallelogram whose sides are equal to the parts, and whose angles are equal to those of the rhombus.



On AB describe a rhombus.

In other respects construct the figure as in Prop V., and the proof is nearly the same with that in Prop. V.

*The following GEOMETRICAL PROBLEMS may be solved by simple applications of the Propositions in the First Two Books of Euclid. If the student prefer going on to the Third Book at once, he may do so ; but he will find his progress quicker, if he from time to time practises himself in these or any other Geometrical Problems.*

IF a perpendicular be drawn bisecting a given straight line, any point in this perpendicular is at equal distances, and any point without the perpendicular is at unequal distances from the extremities of the line.

---

Any side of a triangle is greater than the difference between the other two sides.

---

The difference of the angle at the base of any triangle is double the angle contained by a line drawn from the vertex perpendicular to the base, and another bisecting the angle at the vertex.

---

To find a point within a triangle which is equidistant from the three angles.

---

To describe a square which shall be equal to the difference of two squares, whose sides are given.

---

To describe a rectangular parallelogram which shall be equal to a given square, and have its adjacent sides together equal to a given line.

---

To describe a square which shall be equal to the sum of any number of given squares.

---

To inscribe a square in a given right-angled isosceles triangle.

---

To inscribe a square in a given quadrant of a circle.

---

If straight lines be drawn from the angles of a triangle through any point, either within or without the triangle, to meet the sides, and the lines joining these points of intersection and the sides of the triangle be produced to meet; the three points of concourse will be in the same straight line.

---

If the opposite sides or opposite angles of a quadrilateral figure be equal, the figure will be a parallelogram.

---

The two triangles, formed by drawing straight lines from any point within a parallelogram to the extremities of two opposite sides, are together half of the parallelogram.

---

If two sides of a trapezium be parallel, its area is equal to half that of a parallelogram whose base is the sum of those two sides, and altitude the perpendicular distance between them.

---

If from one of the acute angles of a right-angled triangle, a line be drawn to the opposite side; the squares of that side, and the line so drawn, are together equal to the squares of the segment adjacent to the right angle and of the hypotenuse.

---

In any triangle, if a line be drawn from the vertex at right angles to the base; the difference of the squares of the sides is equal to the difference of the squares of the segments of the base.

---

In any triangle, if a line be drawn from the vertex bisecting the base; the sum of the squares of the two sides of the triangle is double the sum of the squares of the bisecting line and of half the base.

---

## THE ELEMENTS OF EUCLID.

Given one angle, a side adjacent to it, and the difference of the other two sides; to construct the triangle.

---

Given one angle, a side opposite to it, and the difference of the other two sides; to construct the triangle.

---

If the three sides of a triangle be bisected, the perpendiculars drawn to the sides at the three points of bisection, will meet in the same point

---

To trisect a given triangle from a given point within it.

---

To determine a point within a given triangle from which lines drawn to the several angles will divide the triangle into three equal parts.

---

Of all triangles having the same vertical angle, and whose bases pass through a given point, the least is that whose base is bisected in the given point.

---

The sum of the sides of an isosceles triangle is less than the sum of the sides of any other triangle on the same base and between the same parallels.

---

If from the extremity of the base of an isosceles triangle, a line equal to one of the sides be drawn to meet the opposite side; the angle formed by this line and the base produced, is equal to three times either of the equal angles of the triangle.

THE END.

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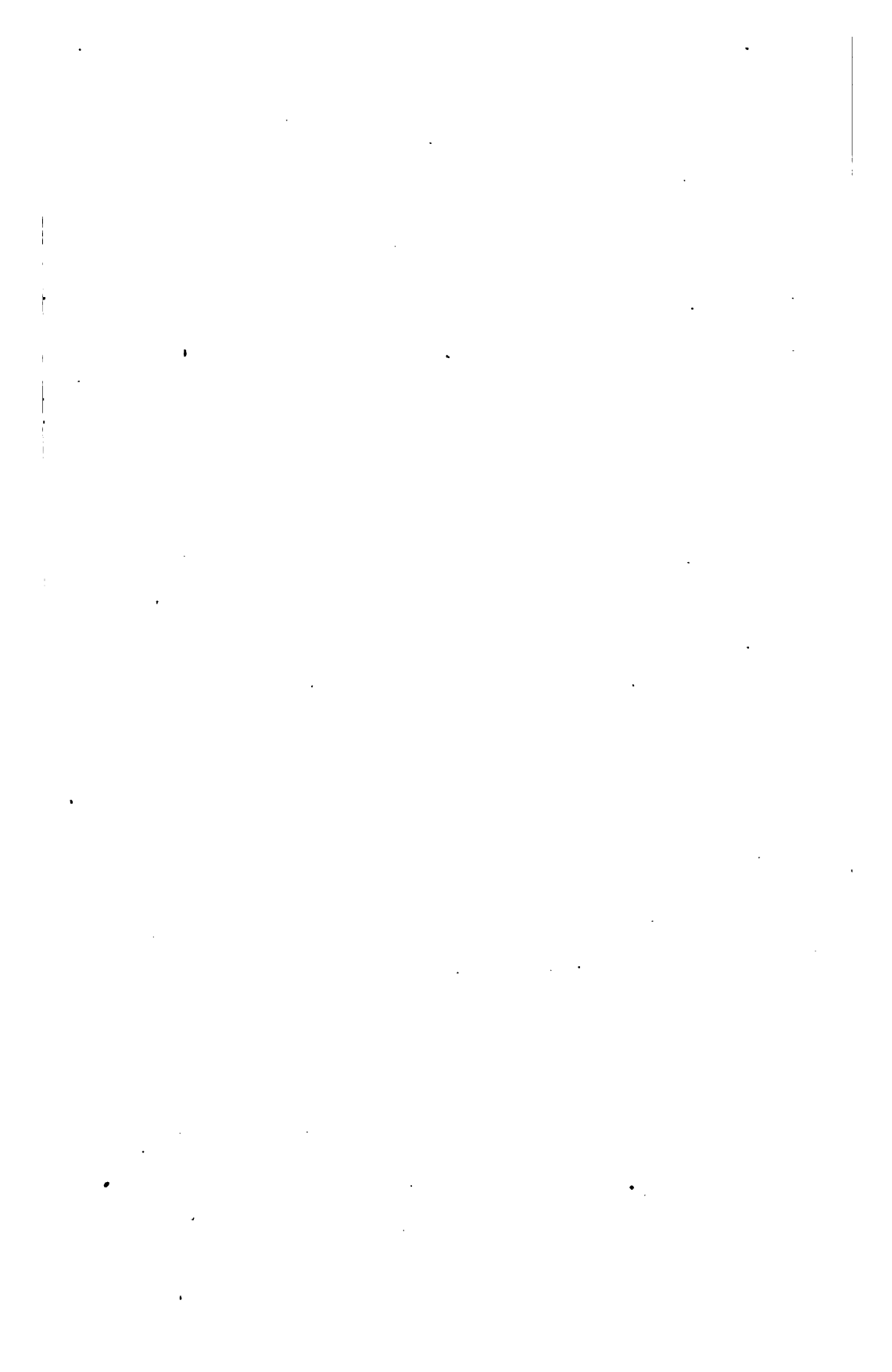
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